Prof. Dr. E. Spodarev / W. Karcher SS 2008
17/4/2008

Homework assignment for Risk Theory - #1

(Due Thursday, 24/4/2008, 10:15 a.m., H3)

1. Let X be a discrete random variable on a probability space $(\Omega, \mathcal{F}, \mathbb{P})$ with values in $\mathbb{N}$ and let $p := \mathbb{P}(X = 1) \in (0, 1)$. Show that (6)

$$X \sim \text{Geo}(p) \Leftrightarrow \mathbb{P}(X - n_0 \geq k | X > n_0) = \mathbb{P}(X \geq k) \quad \forall k \in \mathbb{N}, n_0 \in N_0.$$

2. Both the Pareto-distribution and the Weibull-distribution play an important role in non-life insurance. The density of the Pareto-distribution Par(α, c) with parameters $\alpha, c > 0$ is given by

$$f_{\text{Par}(\alpha,c)}(x) = \frac{\alpha}{c}\left(\frac{c}{x}\right)^{\alpha+1} \mathbb{1}_{(c,\infty)}(x),$$

and the density of the Weibull-distribution W(r,c) with parameters $r, c > 0$ is given by

$$f_{\text{W}(r,c)}(x) = rcx^{r-1}e^{-cx^r}\,\mathbb{1}_{[0,\infty)}(x).$$

   (a) Compute the expected value and the variance of the Par(α,c)-distribution. (4)

   (b) Compute the expected value and the variance of the W(r,c)-distribution. (4)

3. The *moment-generating function* $\hat{m}_X$ of a real-valued random variable $X \geq 0$ is given by

$$\hat{m}_X(t) = \mathbb{E}\left(e^{tX}\right),$$

for all $t \in \mathbb{R}$ for which the expected value exists. The *generating function* $\hat{g}_Y$ of a random variable $Y : \Omega \to \mathbb{N}$ is given by

$$\hat{g}_Y(u) = \mathbb{E}\left(u^Y\right)$$

for u in $[-1, 1]$.

   (a) Let $X \sim \text{Exp}(\lambda)$ with $\lambda > 0$. Compute $\hat{m}_X$. (2)

   (b) Let $Y \sim \text{Poi}(\lambda)$ with $\lambda > 0$. Compute $\hat{g}_Y$. (2)

   (c) Let $X_1, ..., X_n$ for $n \geq 2$ be independent random variables. Show that (2)

$$\hat{m}_{X_1+...+X_n}(t) = \prod_{k=1}^{n} \hat{m}_{X_k}(t).$$

   (d) Let $X_1, ..., X_n$ for $n \geq 2$ be independent and identically Exp(λ)-distributed random variables with $\lambda > 0$. What is the distribution of $X_1 + ... + X_n$? (2)

4. Let $X_1 > 0, ..., X_n > 0$ be stochastically independent and identically distributed real-valued random variables. Show that (3)

$$\mathbb{E}\frac{X_1}{X_1 + ... + X_n} = \frac{1}{n}.$$