

### Homework assignment for Risk Theory - #10

(Due Thursday, 3/7/2008, 10:15 a.m., H9)

1. An insurance company wants to estimate the late claims of an insurance portfolio that has been set up in 1998. The estimation has to be based upon the following claim amounts  $S_{ik}$ .

Occurrence year	premium	Claim amounts $S_{ik}$ in run-off year $k$				
		k=1	2	3	4	5
1999	468	150	96	60	8	5
2000	570	180	120	14	9	
2001	630	198	28	21		
2002	98	40	31			
2003	112	42				

- (a) Use the Chain-Ladder Method in order to estimate the expected amount that has to be paid in 2005 for claims that date from the occurrence year 2001. (5)
- (b) Solve (a) using the distribution-free method. (4)
2. Consider the following cumulative claim amounts.

Occurrence year	premium	Cumulative claim amounts $C_{ik}$ in run-off year $k$			
		k=1	2	3	4
2001	120,000	15,000	45,000	70,000	85,000
2002	130,000	15,000	55,000	75,000	
2003	140,000	20,000	65,000		
2004	150,000	20,000			

Suppose that all claims of a certain occurrence year can be settled completely within 4 years. Use the Chain-Ladder Method to

- (a) estimate the expected late claim reserve for the occurrence year 2002. (4)
- (b) estimate the expected amount to be paid in 2005 and 2006 for claims dating from the occurrence year 2003. (5)

3. Let  $X$  be an  $\text{Erl}(n, \lambda)$ -distributed risk with parameters  $n \in \mathbb{N}$  and  $\lambda > 0$ . (4)  
Compute the premium for  $X$  using the

- (a) expected value principle,  
(b) standard deviation principle.

4. For a risk  $X = \sum_{i=1}^N U_i$ , where  $N \sim \text{Poi}(\lambda)$  with  $\lambda > 2$  and  $U_i \sim U(0, 1)$ , let  $\Pi(X)$  denote the premium. The contribution restitution (Beitragsrückgewähr) at the end of the insurance period is (6)

- (i)  $\Pi(X)/2$  if no claim has been reported,  
(ii)  $(\Pi(X) - U)/2$  if a single claim of size  $U$  has been reported.  
(a) Compute  $\Pi(X)$  using the net premium principle.

- (b) Give the value of  $\Pi(X)$  for  $\lambda = 5$ .

5. For a risk with yearly aggregate claim amount  $X$  and distribution (10)

$k$ (Mio) EURO	1	2	3
$\mathbb{P}(X = k)$	0.2	0.5	0.3

an insurance company offers a profit sharing.

In case the risk premium  $\Pi(X)$  exceeds the aggregate claim amount  $X$ , 40% of the difference  $\max(\Pi(X) - X, 0)$  are given back. A self-settlement of claims is not possible.

- (a) Compute the net premium  $\Pi(X)$ .  
(b) For a net premium  $\Pi(X)$  of 2.25 Mio EURO compute the expected value and the standard deviation of the profit after premium refund

$$\max(\Pi(X) - X, 0)0.6 + (\min(\Pi(X), X) - X).$$