Prof. Dr. E. Spodarev / W. Karcher SS 2008
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# Homework assignment for Risk Theory - #11

(Due Thursday, 10/7/2008, 10:15 a.m., H9)

1. A compound risk $X$ is modelled by the number $N$ of

| $k$ | 0 | 1 | 2 |
|---|---|---|---|
| $\mathbb{P}(N=k)$ | 0.5 | 0.3 | 0.2 |

claims, as well as by the claim amounts $U_i$, $i = 1, 2$.

| $u$ EURO | 100 | 500 | 1000 |
|---|---|---|---|
| $\mathbb{P}(U_i = k)$ | 0.5 | 0.3 | 0.2 |

Let the random variables $N$, $U_1$, and $U_2$ be stochastically independent. The insurance company compensates for the first reported claim in full height, but only one third of the claim size of the second reported claim.

(a) Compute the net premium in the case of the insurance holder reporting every claim. (3)

(b) Compute the net premium in the case where the insurance holder reports the first claim only if its amount is 500 or 1000. If the first claim is not reported, the second claim is fully covered (if it occurs). (3)

(c) What will be the behavior of a rational insurance holder if it is possible to report his claims at the end of the insurance period? (3)

2. Let $X$ be exponentially distributed with expectation 100. The risk $X$ (exactly one claim of size $X$ occurs per period) is insured with a gross risk premium (Bruttorisikoprämie) of 80. The retention (Selbstbeteiligung) is 50, and the contribution restitution (in case no claim occurs) is 20. Notice that rational behavior of the insurance holders is always assumed.

(a) Starting from what claim size, the insurance holder will report a claim? (2)

(b) What safety loading (i. e. gross risk premium minus net risk premium) does the insurance company have to allow for? (6)

3. Consider again exercise 4 on sheet # 5. Compute the average steady-state (stationary) premium, if the corresponding premium sizes for classes $P, A, B$, and $D$ are $\pi = (100, 200, 300, 400)$, respectively. (Hint: The average steady-state premium is $(p(P), p(A), p(B), p(D)) \cdot \pi^T$ where $(p(P), p(A), p(B), p(D))$ is the vector of stationary probabilities.) (2)

4. The portfolio of an insurance company consists of two different risks A and B, respectively, where claims occur stochastically independently. For each type of risk, the following table shows the insured sum (IS) and the probabilities $p_0$ (no claim occurs) and $p_1 = 1 - p_0$ (a claim occurs).

| Risk type | IS | $p_0$ | $p_1$ |
|---|---|---|---|
| A | 900 | 0.8 | 0.2 |
| B | 600 | 0.7 | 0.3 |

Beyond that, the probability distribution of a claim $U$ is given in the following table (again separated according to the two risk types).

| Risk type | A | B |
|---|---|---|
| $U = 1/3$ IS | 0.6 | 0.2 |
| $U =$ IS | 0.4 | 0.8 |

(a) Compute the expected value and the coefficient of variation of the aggregate claim amount of the portfolio.

(b) Consider an excess-of-loss reinsurance (Einzelschadenexzedenten-Rückversicherung) with a retention level (Selbstbehalt) of 500. Compute the expected value and the coefficient of variation of the deductible.

(c) Consider now a surplus reinsurance (Summenexzedenten-Rückversicherung) with a retention level of 500. Compute the expected value and the coefficient of variation of the deductible.

(d) Compare the foregoing results with respect to the coefficient of variation.

5. Consider a portfolio of an insurance company that may consist of 100 insurance policies with an insured sum of 4000 Euro and of 300 insurance policies with an insured sum of 10000 Euro. Assume that all claims of a policy occur stochastically independently with a probability of 0.1. The probability that for a certain policy no claims occur is 0.9.

Compare the effects of a surplus reinsurance (with a retention level of 3000 Euro) regarding the expected value and the variance of the deductible. Assume that in case of an actual damage event, a total loss or a partial loss of 50% of the insured sum occurs with a probability of 0.5, respectively.