

Homework assignment for Risk Theory - #2

(Due Wednesday, 30/4/2008, 10:15 a.m., H3)

1. Assume that the duration T of a fire is a random variable with distribution $\text{Exp}(\lambda)$, where $\lambda > 0$ is the parameter. The damage $l(t)$, caused by a fire of duration t , may be given by $l(t) = ae^{bt}$ with $a, b > 0$. What is the distribution of the random variable $l(T)$? (6)

2. (a) Determine the hazard rate for the Pareto distribution. (2)

(b) Determine the asymptotic behavior of the hazard rate $m(x)$ for the gamma distribution as x goes to infinity. (3)

3. Show that

(a) the normal distribution $N(\mu, \sigma^2)$ with parameters $\mu \in \mathbb{R}$ and $\sigma^2 > 0$ has an exponentially bounded tail. (4)

(b) for the tail function \bar{F} of the lognormal distribution $\text{LN}(a, b)$ with parameters $a \in \mathbb{R}$ and $b > 0$ it holds that (6)

$$\exists \beta, c > 0, \quad \text{such that} \quad \bar{F}(x) \leq c \left(\frac{1}{x} \right)^\beta.$$

4. Are the following distributions heavy-tailed or light-tailed? Justify your answer.

(a) The Weibull distribution $W(r, c)$ with the parameters $r, c > 0$. (4)

(b) The Pareto distribution $\text{Par}(\alpha, c)$ with the parameters $\alpha, c > 0$. (3)

(c) The gamma distribution $\Gamma(a, \lambda)$ with the parameters $a, \lambda > 0$. (3)

(d) The Benktander distribution of type I, $\text{BenI}(a, b, c)$, with the parameters $a, b, c > 0$, where $a(a + 1) \geq 2b$ and $ac \leq 1$. (3)

5. Show that if X is a risk with a heavy tail, then (5)

$$\limsup_{x \rightarrow \infty} e^{sx} \bar{F}_X(x) = \infty, \quad \forall s > 0.$$