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SS 2008
30/4/2008

Homework assignment for Risk Theory - #3

(Due Thursday, 8/4/2008, 10:15 a.m., H3)

1. Prove the following equivalence: (5)

$$F(x) = \left(1 - e^{-\lambda x}\right) \cdot \mathbb{1}_{[0,\infty)}(x) \quad \Longleftrightarrow \quad \mu_F(x) = \frac{1}{\lambda}$$

2. (a) Determine the mean residual lifetime for the Pareto distribution. (2)

   (b) Determine the asymptotic behavior of the mean residual lifetime for the gamma distribution as x goes to infinity. (2)

3. (a) Prove that (3)

$$\mu_F(x) = \int_x^\infty e^{-\int_x^t m(y)dy} dt, \quad x \geq 0.$$

   where $m(\cdot)$ is the hazard rate function.

   (b) Show that if $m(x) \geq \lambda$, then $\mu_F(x) \leq \frac{1}{\lambda}$ and that if $m(x) \leq \lambda$, then $\mu_F(x) \geq \frac{1}{\lambda}$. (3)

   (c) Show that if $m(x) \geq \lambda$, then $\overline{F}(x) \leq e^{-\lambda x}$ and that if $m(x) \leq \lambda$, then $\overline{F}(x) \geq e^{-\lambda x}$. (3)

4. (a) Show that if X is a continuous risk with density f_X, then (2)

$$\frac{1}{m(x)} = \int_0^\infty \frac{f_X(x+y)}{f_X(x)} dy.$$

   (b) Determine if the gamma distribution has a decreasing or an increasing hazard rate function. (2)

5*. On the homepage, a sample of 10.000 claims from a portfolio of storm insurance contracts is provided.

   (a) Plot the mean residual hazard function (restrict the x-axis from 0 to 20.000 and the y-axis from 4.000 to 15.000). Is the sample rather heavy-tailed or light-tailed? (2)

   (b) Generate quantile plots for the gamma ($a = 1$, $\lambda = \frac{1}{4429}$), the lognormal ($\mu = 8$, $\sigma^2 = 1$) and the Weibull distribution ($r = 1$, $c = \frac{1}{4792}$) and discuss the resulting graphs. (4)

   It is recommended to use R for solving this exercise. Please hand in both your code and the plots. Hints on using R can be found on the homepage.