Prof. Dr. E. Spodarev / W. Karcher SS 2008
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Homework assignment for Risk Theory - #4

(**Due Thursday, 21/5/2008, 8:15 a.m., H7**)

1. On the homepage, a sample of monthly claim counts from a portfolio of insurance contracts is provided.

   (a) Based on the index of dispersion, assess whether the homogeneous Poisson process is an appropriate model for the monthly number of claim counts. (2)

   (b) Assume that it is known that the data comes from a mixed Poisson process with $\mathbb{P}(\Lambda = 1) = p$ and $\mathbb{P}(\Lambda = 10) = 1 - p$. Use the estimated value of the index of dispersion from part (a) to determine p. (4)

2. Show that a real-valued stochastic process $\{X(t), t \geq 0\}$ with independent increments has stationary increments if for all $t \geq 0$ the distribution of the (one-dimensional) random variable $X(t+h) - X(h)$ does not depend on $h \geq 0$. (6)

3. Prove the following part of Theorem 3.2.1.

   (a) Statement 1) $\Longrightarrow$ Statement 2) (6)

   (b) Statement 2) $\Longrightarrow$ Statement 3) (6)

   (c) Statement 5) $\Longrightarrow$ Statement 1) (6)

4. Suppose that the number of claims is a mixed Poisson variable with mean 10.000 and standard deviation 1.000. Find the standard deviation of the mixing variable. (4)

5. Let $\{N(t), t \geq 0\}$ be a Pascal process with parameters $a, b > 0$.

   (a) Show that (4)

   $$\hat{g}_{N(t)}(s) = \left(\frac{b}{b + t(1-s)}\right)^a, \quad s \in (-1, 1), t \geq 0.$$

   (b) Show that (6)

   $$p_k(t) = \binom{a+k-1}{k} \left(\frac{t}{t+b}\right)^k \left(\frac{b}{t+b}\right)^a,$$

   i.e. $N(t)$ follows a negative binomial distribution with parameters a and $\frac{t}{t+b}$.