Prof. Dr. E. Spodarev / W. Karcher SS 2008
21/5/2008

Homework assignment for Risk Theory - #5

(Due Thursday, 29/5/2008, 10:15 a.m., H9)

1. Consider the (random) number N of forest fires in British Columbia, Canada in July. Assume that N is a mixed Poisson-distributed random variable with the long-term average $\mathbb{E}N = 20$. The following table classifies weather conditions into 5 classes and shows the distribution of the mixing variable.

Weather type i	λ_i	p_i
Very dry	60	0.05
Dry	35	0.20
Normal	16	0.40
Wet	12	0.25
Very wet	6	0.10

Table 1

(a) Check that the expected value of N is indeed equal to the expected value of the mixing variable. (4)

(b) Compute the values of the cdf $F_N(x)$ for $x = 10, 14, 20, 30, 40, 60$. Consider both the case where N is mixed Poisson-distributed as given above and the case where N follows a Poisson distribution with parameter 20. You may use R for this exercise. Hints can be found online. (5)

2. Consider a bonus-malus system with one malus class M and two bonus classes B_1 and B_2. The insurance collective may consist of two types of risks, R_1 and R_2, respectively, which differ only by the distribution of their claim numbers. Table 2 shows probabilities for the number of claims per year. If a claim-free year occurs, the policy holder is reclassified into the superior class in the following year. If he happens to be in class B_2 already, he remains in that class.

Risk type	Number of claims 0	1	2
R_1	0.7	0.2	0.1
R_2	0.5	0.3	0.2

Table 2

If one claim or two claims occur, the insurance holder is downgraded (no matter what the claim size is) one class or two classes, respectively, in the following year. However, the maximal class to be downgraded to is class M.

The expected size of a claim is 500 EUR. All claims occur independently. In exercise (b) and (c), we assume that a policy holder starts in class M in the first year.

(a) Compute the transition matrices for the two risks. (3)

(b) What is the probability of a policy holder of risk type R_2 to be in class B_1 after 3 years? (5)

(c) Assume that a policy holder is chosen randomly from a portfolio where 60% of all policies are of risk type R_1 and 40% of all policies are of risk type R_2. What is the probability of this policy holder to be in class B_1 after 2 years? (5)

3. Consider a bonus-malus system with 3 classes A, B, and C and classification diagram given in Table 3. For the number of claims N per year we have $\mathbb{P}(N = 0) = \frac{5}{9}$, $\mathbb{P}(N = 1) = \frac{1}{9}$, and $\mathbb{P}(N > 1) = \frac{1}{3}$.

	A	B	C
A	0	1	> 1
B	0	1	> 1
C	–	0	> 0

Table 3

(a) What is the probability of a policy holder to be in class A after 2 years under the assumption that he is in class A at the beginning of the first year? (5)

(b) Use the transition matrix (5)

$$P = (p_{ij})_{i,j \in \{A,B,C\}}$$

in order to compute the stationary distribution

$$(p(A), p(B), p(C))$$

i.e. the solution of the system of equations

$$(p(A), p(B), p(C))P = (p(A), p(B), p(C))^T.$$

(c) How do you have to choose $\mathbb{P}(N = 0)$, $\mathbb{P}(N = 1)$, and $\mathbb{P}(N > 1)$ in order to get that $p(A) = p(B) = p(C)$? (5)

4. Consider a bonus-malus system with classes P (perfect), A (average), B (bad), and D (dangerous). Given are the following transition rules.

   1. From P to P if 0 claims occur and 1 claim occurs, to A if 2 claims occur, to B if 3 claims occur and to D if more than 3 claims occur.
   2. From A to P if 0 claims occur, to A if 1 claim occurs, to B if 2 claims occur and to D if more than 2 claims occur.
   3. From B to A if 0 claims occur, to B if 1 claim occurs, and to D if more than 1 claim occurs.
   4. From D to B if 0 claims occur and to D if more than 0 claims occur.

(a) Construct the transition matrix using the probabilities 0.6 that 0 claims occur, 0.2 that 1 claim occurs, 0.1 that 2 claims occur, and 0.1 that 3 claims occur. (3)

(b) Compute the vector $(p(A), p(B), p(C))$ of stationary probabilities. (4)