Homework assignment for Risk Theory - #7

(Due Thursday, 12/6/2008, 10:15 a.m., H9)

1. Let $X = \sum_{i=1}^{N} U_i$ be a Poisson compound risk with $\mathbb{E}U_i^2 < \infty$. Prove the following central limit theorem. (6)

$$\frac{X - \mathbb{E}X}{\sqrt{\mathrm{Var}X}} \xrightarrow{d} \mathrm{N}(0,1), \quad \lambda \to \infty.$$

2. Consider again the fire insurance on buildings of sheet #6. Table 1 and Table 2 provide information about the number of claims and claim sizes.
   (a) Estimate the expected value and the variance of the total claim amount X. (4)
   (b) Determine the reserve capital that is required such that the total claim amount is covered with a probability of at least 95%. Use the Tschebyschew inequality for this purpose. (4)

k	Number of policies with k claims
0	103705
1	11632
2	1767
3	255
4	44
5	6
6	2
> 7	0
	$\Sigma = 117411$

Table 1

Claim size in the interval (in 100 €)	Number of claims	Average claim size (in 100 €)
$(0, 50]$	51	39
$(50, 100]$	118	72
$(100, 150]$	115	120
$(150, 200]$	77	179
$(200, 300]$	204	249
$(300, 500]$	583	399
$(500, 800]$	1278	647
$(800, 1000]$	818	898
$(1000, 2000]$	3569	1401
$(2000, 5000]$	6056	3009
$(5000, 10000]$	2162	6729
$(10000, 20000]$	807	13511
$(20000, 50000]$	251	27590
$(50000, 100000]$	60	69426
	$\Sigma = 16149$	3817

Table 2

3. Determine the cumulative distribution function of the total claim amount for any compound distribution with exponentially distributed claim sizes. (6)

4. An individual loss distribution is normal with $\mu = 100$ and $\sigma^2 = 9$. The distribution for the number of claims, N, is given in Table 3. Determine the probability that aggregate claims exceed 100. (6)

n	$\mathbb{P}(N = n)$
0	0.5
1	0.2
2	0.2
3	0.1

Table 3

Give a reason why modelling losses with the normal distribution (in particular with $\mu = 100$ and $\sigma^2 = 9$) may be reasonable although negative values are possible.