Homework assignment for Risk Theory - #7
(Due Thursday, 12/6/2008, 10:15 am, H9)

1. Let \( X = \sum_{i=1}^{N} U_i \) be a Poisson compound risk with \( \mathbb{E} U_i^2 < \infty \). Prove the following central limit theorem.

\[
\frac{X - \mathbb{E}X}{\sqrt{\text{Var}X}} \xrightarrow{d} N(0, 1), \quad \lambda \to \infty.
\]

2. Consider again the fire insurance on buildings of sheet #6. Table 1 and Table 2 provide information about the number of claims and claim sizes.

   (a) Estimate the expected value and the variance of the total claim amount \( X \).

   (b) Determine the reserve capital that is required such that the total claim amount is covered with a probability of at least 95%. Use the Tchebycheff inequality for this purpose.

   Number of policies
   
   \[
   \begin{array}{c|c}
   k & \text{Number of policies} \\
   \hline
   0 & 103705 \\
   1 & 11632 \\
   2 & 1767 \\
   3 & 255 \\
   4 & 44 \\
   5 & 6 \\
   6 & 2 \\
   > 7 & 0 \\
   \hline
   \end{array}
   \]

   \( \Sigma = 117411 \)

   Table 1

   Claim size
   
   \[
   \begin{array}{c|c|c}
   \text{in the interval} & \text{Number of claims} & \text{Average claim size} \\
   \hline
   \text{(in 100 €)} & \text{(in 100 €)} & \\
   \hline
   (0, 50] & 51 & 39 \\
   (50, 100] & 118 & 72 \\
   (100, 150] & 115 & 120 \\
   (150, 200] & 77 & 179 \\
   (200, 300] & 204 & 249 \\
   (300, 500] & 583 & 399 \\
   (500, 800] & 1278 & 647 \\
   (800, 1000] & 818 & 898 \\
   (1000, 2000] & 3569 & 1401 \\
   (2000, 5000] & 6056 & 3009 \\
   (5000, 10000] & 2162 & 6729 \\
   (10000, 20000] & 807 & 13511 \\
   (20000, 50000] & 251 & 27590 \\
   (50000, 100000] & 60 & 69426 \\
   \hline
   \end{array}
   \]

   \( \Sigma = 16149 \quad 3817 \)

   Table 2

3. Determine the cumulative distribution function of the total claim amount for any compound distribution with exponentially distributed claim sizes.

4. An individual loss distribution is normal with \( \mu = 100 \) and \( \sigma^2 = 9 \). The distribution for the number of claims, \( N \), is given in Table 3. Determine the probability that aggregate claims exceed 100.

   \[
   n \quad P(N = n)
   \]

   \[
   \begin{array}{c|c}
   \hline
   0 & 0.5 \\
   1 & 0.2 \\
   2 & 0.2 \\
   3 & 0.1 \\
   \hline
   \end{array}
   \]

   Table 3

Give a reason why modelling losses with the normal distribution (in particular with \( \mu = 100 \) and \( \sigma^2 = 9 \)) may be reasonable although negative values are possible.