Prof. Dr. E. Spodarev / W. Karcher SS 2008
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Homework assignment for Risk Theory - #8

(Due Thursday, 19/6/2008, 10:15 a.m., H9)

1. Let $X = \sum_{i=1}^{N} U_i$ be the aggregate claim amount in the collective model, where $N \sim \text{Geo}(p)$ and $U \sim \text{Exp}(\delta)$. Let $\overline{F}_X(x) = \mathbb{P}(X > x)$.
   (a) Show that (6)
   $$\overline{F}_X(x) = pe^{-(1-p)\delta x}, \quad x \geq 0.$$
   (b) Determine the net stop-loss premium of the reinsurer if the retention limit of the primary insurer is $b > 0$. (2)

2. Consider two risks X and Y, where their respective distributions are given by
   - $\mathbb{P}(X = 0) = 0.9$ and $\mathbb{P}(X > t) = 0.1e^{-t}$.
   - $\mathbb{P}(Y = 0) = 0.8$ and $\mathbb{P}(Y > t) = 0.2(1+t)^{-3}$.

   (a) Show that both risks have the same expectation and variance. (4)
   (b) Compare the two risks using the semivariance (4)
   $$\mathbb{E}\left[(X - \mathbb{E}X)_+\right]^2 = \mathbb{E}(X - \mathbb{E}X)^2 \mathbb{1}_{\{X > \mathbb{E}X\}},$$
   and decide which of these risks is the more dangerous one.

   Hint: In part (b), you may use
   $$\int_{0.1}^{\infty} (x - 0.1)^2 \frac{3}{(1+x)^4} dx = \frac{10}{11}.$$

3. Suppose that for the aggregate claim amount $X = \sum_{i=1}^{N} U_i$ there is an interval with no aggregate probability, i. e. $\mathbb{P}(a < X < b) = 0$, $a < b$. Show that for $a \leq d \leq b$, (5)
   $$\mathbb{E}\left[(X - d)_+\right] = \frac{b-d}{b-a}\mathbb{E}\left[(X - a)_+\right] + \frac{d-a}{b-a}\mathbb{E}\left[(X - b)_+\right].$$
   That is, the net stop-loss premium can be calculated via linear interpolation.

4. A reinsurer pays aggregate claim amounts in excess of d, and in return it receives a stop-loss premium $\mathbb{E}\left[(X - d)_+\right]$. You are given $\mathbb{E}\left[(X - 100)_+\right] = 15$, $\mathbb{E}\left[(X - 120)_+\right] = 10$, and the probability that the aggregate claim amounts are greater than 80 and less than or equal to 120 is 0. Determine the probability that the aggregate claim amounts are less than or equal to 80. (4)

5. The following data is given for a portfolio of insurance contracts. (5)

Benefit amount	Number of policies	Probability of 1 claim	Probability of no claim
Exp $\left(\frac{1}{500}\right)$	3	0.025	0.975

   All claims are mutually independent. The insurance company uses the standard deviation principle to calculate the premium, i. e. $\Pi(X) = \mathbb{E}X + K\sqrt{\text{Var}(X)}$, $K > 0$. Assume that the initial capital u is equal to 0. Which of the following values of K guaranties the solvability of the portfolio with probability 0.95?
   - $K = 0.5$
   - $K = 0.7$
   - $K = 0.9$

   Show all of your calculations.