Prof. Dr. E. Spodarev / W. Karcher SS 2008
19/6/2008

Homework assignment for Risk Theory - #9

(Due Thursday, 26/6/2008, 10:15 a.m., H9)

1. Let $N \sim \text{Poi}(\lambda)$. Show the following formulae for the n^{th} moment of the k^{th} largest claim in the collective model $X = \sum_{i=1}^{N} U_i$.

   (a) For $U_i \sim \text{Exp}(\delta)$ it holds that (5)

   $$\mathbb{E}(U^n_{(N-k+1)}) = \frac{\lambda^k}{\delta^n (k-1)!} \int_0^1 e^{-\lambda x} (-\log x)^n x^{k-1} dx, \quad n \in \mathbb{N}.$$

   (b) For $U_i \sim \text{Par}(\alpha, 1)$ it holds that (5)

   $$\mathbb{E}(U^n_{(N-k+1)}) = \frac{\lambda^k}{(k-1)!} \int_0^1 e^{-\lambda x} x^{k-1-n/\alpha} dx, \quad n < \alpha k, \quad n \in \mathbb{N}.$$

2. Within a portfolio of a certain health insurance company, the claim size distribution for two different diseases A and B and for two different age groups (20-40 years and 40-60 years, respectively) is examined. Assume that 4 insurance policies are in each class. The probability that someone contracts a disease (i. e. the probability of a claim) is 0.015 for the younger age group and 0.02 for the elder age group. The conditional claim size distributions for disease A and B, respectively, are given in the following vectors, where the entry $p_k^{(i)}$ is the probability that the claim size in class i (i. e. for disease A or B) takes value k.

$$\begin{pmatrix} p_1^{(A)} \\ p_2^{(A)} \\ p_3^{(A)} \\ p_4^{(A)} \\ p_5^{(A)} \\ p_6^{(A)} \\ p_7^{(A)} \\ p_8^{(A)} \\ p_9^{(A)} \end{pmatrix} = \begin{pmatrix} 1/9 \\ 1/9 \\ 1/9 \\ 1/9 \\ 1/9 \\ 1/9 \\ 1/9 \\ 1/9 \\ 1/9 \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} p_1^{(B)} \\ p_2^{(B)} \\ p_3^{(B)} \\ p_4^{(B)} \\ p_5^{(B)} \\ p_6^{(B)} \\ p_7^{(B)} \\ p_8^{(B)} \\ p_9^{(B)} \end{pmatrix} = \begin{pmatrix} 0.02 \\ 0.15 \\ 0.1 \\ 0.05 \\ 0.1 \\ 0.05 \\ 0.1 \\ 0.05 \\ 0.2 \end{pmatrix}$$

   (a) Describe the model behind De Pril's algorithm and its generalization (Theorem 4.4.1) for the situation described above. (4)

   (b) Implement the generalized algorithm of De Pril and calculate the probability function $\{p_k\}_{k=1,\ldots,m}$ of the aggregate claim amount. (12)

3. Consider again exercise # 1(a) of assignment # 8. (10)
   Discretize U by $U_h = h\lfloor \frac{U}{h} \rfloor$ for $h = 0.01$. Compute $\overline{F}_X(x)$ by means of the discrete Panjer-algorithm for $\delta = 1$, $p = \frac{100}{101}$ and $x = 0.2, 0.4, 0.6, 0.8, 1.0, 2.0$. Compare the results with the results obtained by using the derived formula in exercise # 1(a) of assignment # 8.