1. Let $N \sim \text{Poi}(\lambda)$. Show the following formulae for the $n^{th}$ moment of the $k^{th}$ largest claim in the collective model $X = \sum_{i=1}^{N} U_i$.

   (a) For $U_i \sim \text{Exp}(\delta)$ it holds that
   $$\mathbb{E}(U_n^{(N-k+1)}) = \frac{\lambda^k}{\delta^n (k-1)!} \int_0^1 e^{-\lambda x} (-\log x)^n x^{k-1} dx, \quad n \in \mathbb{N}.$$  

   (b) For $U_i \sim \text{Par}(\alpha, 1)$ it holds that
   $$\mathbb{E}(U_n^{(N-k+1)}) = \frac{\lambda^n}{(k-1)!} \int_0^1 e^{-\lambda x} x^{k-1-n/\alpha} dx, \quad n < \alpha k, \quad n \in \mathbb{N}.$$  

2. Within a portfolio of a certain health insurance company, the claim size distribution for two different diseases A and B and for two different age groups (20-40 years and 40-60 years, respectively) is examined. Assume that 4 insurance policies are in each class. The probability that someone contracts a disease (i.e. the probability of a claim) is 0.015 for the younger age group and 0.02 for the elder age group. The conditional claim size distributions for disease A and B, respectively, are given in the following vectors, where the entry $p_k^{(i)}$ is the probability that the claim size in class $i$ (i.e. for disease A or B) takes value $k$.

   $$\begin{pmatrix}
p_1^{(A)} \\
p_2^{(A)} \\
p_3^{(A)} \\
p_4^{(A)} \\
p_5^{(A)} \\
p_6^{(A)} \\
p_7^{(A)} \\
p_8^{(A)} \\
p_9^{(A)}
\end{pmatrix} = \begin{pmatrix}1/9 \\
1/9 \\
1/9 \\
1/9 \\
1/9 \\
1/9 \\
1/9 \\
1/9 \\
1/9\end{pmatrix} \quad \text{and} \quad \begin{pmatrix}
p_1^{(B)} \\
p_2^{(B)} \\
p_3^{(B)} \\
p_4^{(B)} \\
p_5^{(B)} \\
p_6^{(B)} \\
p_7^{(B)} \\
p_8^{(B)} \\
p_9^{(B)}
\end{pmatrix} = \begin{pmatrix}0.02 \\
0.15 \\
0.1 \\
0.05 \\
0.1 \\
0.05 \\
0.1 \\
0.05 \\
0.2\end{pmatrix}.$$  

   (a) Describe the model behind De Pril’s algorithm and its generalization (Theorem 4.4.1) for the situation described above.  

   (b) Implement the generalized algorithm of De Pril and calculate the probability function $\{p_k\}_{k=1,\ldots,m}$ of the aggregate claim amount.  

3. Consider again exercise #1(a) of assignment # 8. Discretize $U$ by $U_h = h \lfloor \frac{U}{h} \rfloor$ for $h = 0.01$. Compute $F_X(x)$ by means of the discrete Panjer-algorithm for $\delta = 1$, $p = \frac{100}{101}$ and $x = 0.2, 0.4, 0.6, 0.8, 1.0, 2.0$. Compare the results with the results obtained by using the derived formula in exercise #1(a) of assignment # 8.