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Summer term 2010
21th April 2010

Risk Theory

Exercise Sheet 1

Due to: 28th April 2010

Exercise 1 (6 points)

Let $X \sim \mathrm{Exp}(\lambda)$ be exponentially distributed with parameter $\lambda > 0$.

(a) Compute $\mathbb{P}[X > u]$ for $u > 0$.

(b) Compute $\mathbb{P}[a \le X \le b]$ for $0 < a < b < \infty$.

(c) Show that for every $u, v > 0$, $\mathbb{P}[X > u + v | X > u] = \mathbb{P}[X > v]$.

Exercise 2 (6 points)

Let $X \sim \mathrm{W}(r, c)$ be Weibull distributed with parameters $r, c > 0$. The density of X is given by $f_X(t) = rct^{r-1}e^{-ct^r}1_{t\ge 0}$.

(a) Compute $\mathbb{P}[X > u]$ for $u > 0$.

(b) Compute the expectation $\mathbb{E}X$.

(c) Compute the variance $\mathrm{Var}\, X$.

Exercise 3 (6 points)

Let $X \sim \mathrm{Gamma}(\alpha, \lambda)$ be Gamma-distributed with parameters $\alpha > 0$, $\lambda > 0$.

(a) Show that the density of the random variable $Y := e^X$ is given by

$$f_Y(t) = \frac{\lambda^\alpha}{\Gamma(\alpha)} t^{-(\lambda+1)} \log^{\alpha-1}(t) 1_{t>1}.$$

(b) Compute $\mathbb{E}Y$ for $\lambda > 1$ and show that $\mathbb{E}Y = +\infty$ for $\lambda \in (0, 1)$.

(c) Compute $\mathrm{Var}\, Y$ for $\lambda > 2$ and show that $\mathrm{Var}\, Y$ does not exist for $\lambda \in (0, 2)$.

Exercise 4 (6 points)

Let $X_1, \ldots, X_n$ be independent and identically distributed real-valued random variables with $\mathbb{P}[X_1 > 0] = 1$. Show that

$$\mathbb{E}\left[\frac{X_1}{X_1 + \ldots + X_n}\right] = \frac{1}{n}.$$