Risk Theory
Exercise Sheet 10
Due to: 30th June 2010

Exercise 1 *Premium calculation in the presence of a contribution restitution*

Let $S = \sum_{i=1}^{N} U_i$ be the total claim amount in the collective model, where $N \sim$ Poi(λ) and $U_i \sim$ U(0, 1) (uniformly distributed on [0, 1]). The premium, denoted by Π, has to be chosen in such a way that $\Pi = \mathbb{E}(S + R)$, where R is the contribution restitution (Beitragsrückgewähr) defined as follows:

$$R = \begin{cases} \Pi/2, & \text{if no claim has been reported,} \\ (\Pi - U_1)/2, & \text{if a single claim of size } U_1 \text{ has been reported,} \\ 0, & \text{if more than one claim has been reported.} \end{cases}$$

Compute Π and give the value of Π for $\lambda = 5$.

Exercise 2 *Behavior of a rational insurance holder in the presence of retention*

A risk S in the collective model is modeled by the number N of claims as well as by the claim amounts U_i, $i = 1, 2$, where

k	0	1	2
$\mathbb{P}(N = k)$	0.5	0.3	0.2

u Euro	100	500	1000
$\mathbb{P}(U_i = u)$	0.5	0.3	0.2

The insurance company compensates for the first reported claim in full height, but only one third of the claim size of the second reported claim.

(a) Compute the net premium in the case of the insurance holder reporting every claim.

(b) Compute the net premium in the case where the insurance holder reports the first claim only if its amount is 500 or 1000. If the first claim is not reported, the second claim is fully covered (if it occurs).

(c) What will be the behavior of a rational insurance holder at the end of the insurance period knowing the number of his claims and the claim sizes if it is possible not to report one of the claims. The answer should depend on the number of claims and the claim sizes.

Exercise 3 *Behavior of a rational insurance holder in the presence of retention and contribution restitution*

Let X be exponentially distributed with expectation 100. The risk X (exactly one claim of size X occurs per period) is insured with a gross risk premium (Bruttorisikoprämie) of 80. The retention (Selbstbeteiligung) is 50, and the contribution restitution (in case no claim occurs) is 20. Notice that rational behavior of the insurance holders is always assumed.

(a) Starting from what claim size, the insurance holder will report a claim?

(b) Compute the safety loading (i. e. gross risk premium minus net risk premium).

Definitions.

Retention is the amount of money that an insurance holder has to pay by himself when he reports the claim. For example, in Problem 3, the insurance company pays $\max(X - 50, 0)$ to the insurance holder if a claim is reported.

Contribution restitution is the amount of money an insurance holder obtains when the number of claims he reports in an insurance period is not too large. For example, in Problem 3, the insurance company pays 20 to the insurance holder if no claim is reported.

The net risk premium is the expected value of the payments of the insurance company from a single contract. Note that it is not $\mathbb{E}X$ in Problem 3.

Exercise 4 *Normal-Power-Approximation*

The distribution of the total claim size in the individual model S may be approximated by the Normal-Power-Approximation (here, $\mu = \mathbb{E}S$, $\sigma^2 = \operatorname{Var} S$, $\gamma = \mathbb{E}(S - \mu)^3/\sigma^3$):

$$\mathbb{P}(S \leq t) \approx \Phi\left(\frac{1}{\gamma} \cdot \left(\sqrt{\gamma^2 + 6\gamma \cdot \frac{t - \mu}{\sigma} + 9} - 3\right)\right).$$

(a) Determine the median of the approximation (depending on μ, σ and γ).

(b) Determine the 95%- and the 5%-quantile of the approximation if $\mu = 300$, $\sigma = 7$ and $\gamma = 1$.