# Risk Theory
## Exercise Sheet 11
### Due to: 7th July 2010

**Exercise 1** (6 points) *Claim reserves*

An insurance company wants to estimate the late claims of an insurance portfolio that has been set up in 1998. The estimation has to be based upon the following claim amounts $S_{ik}$.

| Occurrence year | premium | Claim amounts $S_{ik}$ in run-off year $k$ | | | | |
|---|---|---|---|---|---|---|
| | | k=1 | 2 | 3 | 4 | 5 |
| 1999 | 468 | 150 | 96 | 60 | 8 | 5 |
| 2000 | 570 | 180 | 120 | 14 | 9 | |
| 2001 | 630 | 198 | 28 | 21 | | |
| 2002 | 98 | 40 | 31 | | | |
| 2003 | 112 | 42 | | | | |

(a) Use the Chain-Ladder Method in order to estimate the expected amount that has to be paid in 2005 for claims that date from the occurrence year 2001.

(b) Solve (a) using the distribution-free method.

**Exercise 2** (6 points) *Premium calculation*

Let $X$ be an $\Gamma(n, \lambda)$-distributed risk with parameters $n \in \mathbb{N}$ and $\lambda > 0$. Compute the premium for $X$ using the

(a) expected value principle,

(b) standard deviation principle.

**Exercise 3** (6 points) *Reinsurance in the collective model*

The portfolio of an insurance company consists of two different risks A and B, respectively, where claims occur stochastically independently. For each type of risk, the following table shows the insured sum (IS) and the probabilities $p_0$ (no claim occurs) and $p_1 = 1 - p_0$ (a claim occurs).

| Risk type | IS | $p_0$ | $p_1$ |
|-----------|-----|------|------|
| A | 900 | 0.8 | 0.2 |
| B | 600 | 0.7 | 0.3 |

Beyond that, the probability distribution of a claim $U$ is given in the following table (again separated according to the two risk types).

| Risk type | A | B |
|-----------|-----|-----|
| $U = 1/3$ IS | 0.6 | 0.2 |
| $U = $ IS | 0.4 | 0.8 |

(a) Compute the expected value and the coefficient of variation of the aggregate claim amount of the portfolio (collective model).

(b) Consider an excess-of-loss reinsurance (Einzelschadenexzedenten-Rückversicherung) with a retention level (Selbstbehalt) of 500. Compute the expected value and the coefficient of variation of the deductible.

(c) Consider now a surplus reinsurance (Summenexzedenten-Rückversicherung) with a retention level of 500. Compute the expected value and the coefficient of variation of the deductible.

(d) Compare the foregoing results with respect to the coefficient of variation.

**Exercise 4** (6 points) *Reinsurance in the individual model*

Consider a portfolio of an insurance company that may consist of 100 insurance policies with an insured sum of 4000 Euro and of 300 insurance policies with an insured sum of 10000 Euro. Assume that all claims of a policy occur stochastically independently with a probability of 0.1. The probability that for a certain policy no claims occur is 0.9.

Compare the effects of a surplus reinsurance (with a retention level of 3000 Euro) regarding the expected value and the variance of the deductible. Assume that in case of an actual damage event, a total loss or a partial loss of 50% of the insured sum occurs with a probability of 0.5, respectively.