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Risk Theory

Exercise Sheet 12

Due to: 14th July 2010

Exercise 1 (20 points) *Bühlmann credibility*

(a) Assume that the number of claims made by an individual insured in a year has a Poisson distribution with mean θ. The prior distribution for Θ is gamma with parameters $\alpha = 1$ and $\beta = 1.2$. Three claims are observed in year 1, and no claims are observed in year 2. Estimate the number of claims in year 3.

(b) Assume that the annual number of claims for an individual policyholder has mean θ and variance λ. The prior distribution for Θ is uniform on the interval $[0.5, 1.5]$. The prior distribution for Λ is exponential with mean 1.25. A policyholder is selected at random and observed to have no claims in year 1. Estimate the number of claims in year 2 for the selected policy holder.

(c) Two risks have the following claim size distribution:

Amount of claim	Probability of claim amount for risk 1	Probability of claim amount for risk 2
250	0.5	0.7
2500	0.3	0.2
60000	0.2	0.1

Risk 1 is twice as likely to be observed as risk 2. A claim of 250 is observed. Determine the Bühlmann credibility estimate of the second claim amount from the same risk.

(d) The annual number of claims on a given policy with risk parameter θ has a geometric distribution with parameter $1/(1+\theta)$. The prior distribution of Θ has the density function

$$f(\theta) = \begin{cases} \frac{\alpha}{(\theta+1)^{(\alpha+1)}}, & \theta > 0, \\ 0, & \text{otherwise}, \end{cases}$$

where $\alpha > 2$. A randomly selected policy has x claims in year 1. Determine the Bühlmann credibility estimate of the number of claims for the selected policy in year 2.

(e) The annual number of claims for an insured has the probability function

$$p(x) = \binom{3}{x} q^x (1-q)^{3-x}, \quad x = 1, 2, 3.$$

The prior probability density function is

$$f(q) = \begin{cases} 2q, & 0 < q < 1, \\ 0, & \text{otherwise.} \end{cases}$$

A randomly chosen insured has zero claims in year 1. Estimate the number of claims in year 2 for the selected insured.

Exercise 2 (6 points) *Claim reserving with inflation*

Consider the following cumulative claim amounts.

		Cumulative claim amounts C_{ik} in run-off year k			
Occurrence year	(premium)	k=1	2	3	4
2001	(90158)	11270	36063	58790	73790
2002	(107438)	12397	48761	68761	
2003	(127273)	18182	63182		
2004	(150000)	20000			

Suppose that all claims of a certain occurrence year can be settled completely within 4 years. In the past, price increases of 10% per year of the claim amounts occurred. Therefore, we have to assume that these price increases will continue in future years. Notice that all numbers given in this exercise have been rounded to integer numbers.

Use the Chain-Ladder method to

(a) estimate the expected late claim reserve for the occurrence year 2002.

(b) estimate the expected amount to be paid in 2005 and 2006 for claims dating from the occurrence year 2003.

All computations should be done on the basis of inflation-adjusted data with respect to the base year 2004.