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Risk Theory

Exercise Sheet 5

Due to: 26th May 2010

Exercise 1 (6 points) *A generalization of the Weibull distribution*

(a) For $B, \alpha, c, r > 0$, define $f(t) = Bt^{\alpha}e^{-ct^r}1_{t>0}$. Find a value of B (depending on α,c,r) which makes f a valid density function.

(b) Let X be a random variable with density f as above. Show that the tail of X satisfies $\mathbb{P}[X > t] \sim (B/cr)t^{\alpha-r+1}e^{-ct^r}$ as $t \to \infty$.

Remark. We write $f(t) \sim g(t)$ as $t \to \infty$ if $\lim_{t\to\infty} \frac{f(t)}{g(t)} = 1$.

Exercise 2 (6 points) *Some distributions with regularly varying tails*

(a) Let X be a random variable with Lévy distribution, i.e., the density of X is given by $f_X(t) = (2\pi)^{-1/2}t^{-3/2}e^{-1/(2t)}1_{t>0}$. Show that $\mathbb{P}[X > t] \sim 2(2\pi)^{-1/2}t^{-1/2}$ as $t \to +\infty$.

(b) Let X be a Cauchy random variable, i.e., the density of X is $f_X(t) = \frac{1}{\pi}\frac{1}{1+t^2}$. Show that $\mathbb{P}[X > t] \sim \frac{1}{\pi t}$ as $t \to +\infty$.

Exercise 3 (6 points) *Variation of the number of claims in a mixed Poisson process*

Let $(N(t))_{t\geq 0}$ be a mixed Poisson process with mixing parameter Λ, where Λ is a random variable taking positive values with probability 1. Assume that $\mathbb{E}\Lambda^2 < \infty$. Show that $\mathrm{Var}\, N(t) = t\mathbb{E}\Lambda + t^2\mathrm{Var}\,\Lambda$.

Exercise 4 (6 points) *First claim arrival in the Pólya–Lundberg process*

Show that in the Pólya–Lundberg process with parameters $\alpha > 0$ and $\gamma > 0$, the distribution of the first claim arrival time T_1 is given by

$$f_{T_1}(t) = \frac{\gamma}{\alpha}\left(\frac{\alpha}{\alpha+t}\right)^{\gamma+1} 1_{t\geq 0}.$$

Remark. The Pólya–Lundberg process with parameters $\alpha > 0$ and $\gamma > 0$ is a mixed Poisson process such that the mixing parameter Λ has Gamma(γ, α) distribution.

Exercise 5 (6 points) *Time after the last claim in a Poisson point process*

Let $T_0, T_1, T_2, \ldots$ be a Poisson point process with intensity λ. For $s > 0$ define $N(s) = \sum_{i\in\mathbb{N}} 1_{T_i\leq s}$. Let F_{Y_s} be the distribution function of $Y_s := s - T_{N(s)}$. Compute $F_{Y_s}(z)$, $z \in \mathbb{R}$. Show that $\lim_{s\to+\infty} F_{Y_s}(z) = 1 - e^{-\lambda z}$, $z > 0$.