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Risk Theory

Exercise Sheet 9

Due to: 23th June 2010

Exercise 1 *Expectation and variance in the collective model*

Consider two stochastically independent insurance portfolios (collective model). For the first portfolio, let the number of claims $N \sim \mathrm{Poi}(100)$ and the claim size $X_1 \sim \mathrm{LN}(10, 2)$. For the second portfolio, let the number of claims L be such that $\mathbb{E}L = 1000$, $\operatorname{Var} L = 1200$ and the claim size $Y_1 \sim \mathrm{Exp}(1/5000)$.

(a) Calculate the expected value and the variance of the total claim amount of each portfolio.

(b) Calculate the expected value and the variance of the total claim amount after combining both portfolios.

(c) Provide a lower bound for the capital that the insurance company requires to cover the sum of the total claim amounts of both portfolios with a probability of 99% (use Chebyshev's inequality).

Exercise 2 *Normal approximation for the total claim amount in an individual model*

An insurance company has a portfolio of 1000 independent and identically distributed risks. In each insurance period, a positive claim occurs with probability 0.9 and no claim occurs with probability 0.1 for each risk. If a claim occurs, it is exponentially distributed with expected value 500. The insurance company collects a premium of 70 per period and risk.

Calculate the probability that the total claim amount of the portfolio exceeds the sum of collected premiums using the central limit theorem.

Exercise 3 *Cumulants*

If X is a random variable with logarithmic Laplace transform $g_X(t) := \log \mathbb{E}e^{tX}$, then the numbers $\kappa_n(X) := g^{(n)}(0)$, $n \in \mathbb{N}$, are called the cumulants of X. (Here $g^{(n)}(0)$ is the n-th derivative of g at 0).

(a) Show that $\kappa_1(X) = \mathbb{E}X$, $\kappa_2(X) = \operatorname{Var} X$, $\kappa_3(X) = \mathbb{E}[(X - \mathbb{E}X)^3]$. *Hint:* You may interchange the derivative and the expectation sign without justifying it.

(b) Compute $\kappa_n(X)$ if $X \sim \mathrm{Poi}(\lambda)$. Compute $\kappa_n(X)$ if $X \sim \mathrm{N}(\mu, \sigma^2)$.

(c) Show the following additivity property: if $X_1, \ldots, X_k$ are independent random variables, then $\kappa_n(X_1 + \ldots + X_k) = \sum_{i=1}^k \kappa_n(X_i)$ for every $n \in \mathbb{N}$.

Exercise 4 *Negative binomial distribution is a compound Poisson distribution*

Consider a collective model with the number of claims $N \sim \mathrm{Poi}(\lambda)$ and the claim size X_1 having a logarithmic distribution with parameter $p \in (0, 1)$, that is $\mathbb{P}[X_1 = n] = -\frac{1}{\log(1-p)} \frac{p^n}{n}$, $n \in \mathbb{N}$. Show that the total claim amount $S = X_1 + \ldots + X_N$ satisfies

$$S \sim \widetilde{\mathrm{NB}}\left(-\frac{\lambda}{\log(1-p)}, 1-p\right).$$