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Risk Theory

Exercise Sheet 2

Due to: 5th May 2010

Exercise 1 (6 points) *The mode of the Gamma distribution*

For a random variable X with a continuous density function f_X, the mode is defined as the value $t_{mode} \in \mathbb{R}$ with the property $f_X(t_{\text{mode}}) = \sup_{t\in\mathbb{R}} f_X(t)$, provided such value exists and is unique. The mode can be interpreted as the "most probable" value of X. Let $X \sim \text{Gamma}(\alpha, \lambda)$, where $\alpha > 1$ and $\lambda > 0$. Compute the mode of X. What happens if $\alpha \in (0, 1]$?

Exercise 2 (6 points) *Lévy distribution*

Let $X \sim N(0, 1)$ be a standard normal random variable. Compute the density function of the random variable $Y := 1/X^2$.

Exercise 3 (6 points) *Sums of geometric random variables*

A random variable W is said to have a geometric distribution with parameter $p \in (0, 1]$ (notation: $W \sim \text{Geo}(p)$) if $\mathbb{P}[W = k] = p(1 - p)^{k-1}$ for every $k \in \mathbb{N}$. A random variable T is said to have a negative binomial distribution with parameters $n \in \mathbb{N}$, $p \in (0, 1]$ (notation: $T \sim \text{NB}(n, p)$), if $\mathbb{P}[T = k] = \binom{k-1}{n-1} p^n (1 - p)^{k-n}$ for every $k = n, n + 1, \ldots$.

Let $W_1, W_2 \ldots$ be independent random variables having the geometric distribution with parameter $p \in (0, 1]$. Show that the random variable $T_n := W_1 + \ldots + W_n$ satisfies $T_n \sim NB(n, p)$.

Exercise 4 (6 points) *Convolution property of the negative binomial distribution*

Let $X_1 \sim NB(n_1, p)$ and $X_2 \sim \text{NB}(n_2, p)$ be independent. Show that $X_1 + X_2 \sim \text{NB}(n_1 + n_2, p)$.

Hint: Use the result of Exercise 3.

Exercise 5 (6 points) *Number of claim arrivals in a discrete time model*

Let $W_1, W_2 \ldots$ be independent random variables having the geometric distribution with parameter $p \in (0, 1]$. Define a claim arrival process $T_0, T_1, T_2, \ldots$ by $T_0 = 0$ and $T_n = W_1 + \ldots + W_n$, $n \in \mathbb{N}$. For $s \in \mathbb{N}$, let $N(s) = \sum_{i\in\mathbb{N}} 1_{T_i \le s}$ be the number of claim arrivals up to time s. Show that $N(s) \sim \text{Bin}(s, p)$.

Hint: Compute the probability $\mathbb{P}[N(s) = k]$ for $k = 0, \ldots, s$. Consider the case $k = 0$ first.