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Risk Theory

Exercise Sheet 6

Due to: 2nd June 2010

Exercise 1 (6 points) *Hazard rate and mean excess function of the Pareto distribution*

Let a random variable X be Pareto-distributed with parameters $\alpha > 0, c > 0$, i.e., the distribution function of X is given by $F_X(t) = 1 - (\frac{c}{t})^\alpha$, $t > c$. Compute the hazard rate and the mean excess function of X.

Hint. The mean excess function is given by the formula $e_X(t) = (\bar{F}_X(t))^{-1} \int_t^\infty \bar{F}_X(s)ds$.

Exercise 2 (6 points) *Benktander type II distribution*

A random variable X has Benktander type II distribution with parameters $a > 0$, $b \in (0, 1)$ and $c \in (0, e^{a/b}/a)$ if the distribution function of X is given by

$$F_X(t) = 1 - cat^{b-1}e^{-(a/b)t^b}, \quad t > 1.$$

Compute the hazard rate and the mean excess function of X and show that X is heavy-tailed.

Exercise 3 (6 points) *Gumbel and Fréchet distribution*

(a) A random variable X has Gumbel distribution if the distribution function of X is given by $F_X(t) = e^{-e^{-t}}$ for every $t \in \mathbb{R}$. Show that

$$\bar{F}_X(t) \sim e^{-t}, \quad t \to +\infty.$$

Show that the Gumbel distribution is light-tailed.

(b) A random variable X has Fréchet distribution with parameter $\alpha > 0$ if the distribution function of X is given by $F_X(t) = e^{-t^{-\alpha}}$ for every $t > 0$. Show that the Fréchet distribution is heavy-tailed.

Exercise 4 (6 points) *Limit laws for maxima of i.i.d. random variables*

(a) Let $X_1, X_2, \dots$ be independent random variables having an exponential distribution with $\lambda = 1$. Define $M_n = \max(X_1, \dots, X_n)$. Show that for every $t \in \mathbb{R}$,

$$\lim_{n\to\infty} \mathbb{P}(M_n - \log n \le t) = e^{-e^{-t}}.$$

(b) Let $X_1, X_2, \dots$ be independent random variables having a Pareto distribution with parameters $\alpha > 0$, $c > 0$. Define $M_n = \max(X_1, \dots, X_n)$ Show that for every $t > 0$,

$$\lim_{n\to\infty} \mathbb{P}\left(\frac{M_n}{cn^{1/\alpha}} \le t\right) = e^{-t^{-\alpha}}.$$