Junior-Prof. Dr. Z. Kabluchko
Wolfgang Karcher

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Risk Theory

Exercise Sheet 7

Due to: 9th June 2010

Exercise 1 (6 points) *Max-stable distributions*

(a) Let $X_1, X_2, \dots$ be independent random variables with Gumbel distribution, that is, $\mathbb{P}[X_i \leq t] = e^{-e^{-t}}$ for every $t \in \mathbb{R}$. Show that for every $n \in \mathbb{N}$, the random variable $Z_n := \max(X_1, \dots, X_n) - \log n$ has Gumbel distribution as well.

(b) Let $X_1, X_2, \dots$ be independent random variables having a Fréchet distribution with parameter α, that is, $\mathbb{P}[X \leq t] = e^{-1/t^\alpha}$ for every $t > 0$. Show that the random variable $Z_n := n^{-1/\alpha} \max(X_1, \dots, X_n)$ has Fréchet distribution with parameter α as well.

Exercise 2 (6 points) *Benktander type I distribution*

A random variable X has Benktander type I distribution with parameters $a, b > 0$, where $a(a+1) \geq 2b$, if its tail function is given by

$$\bar{F}_X(t) = a^{-1} t^{-a-1} e^{-b \log^2 t}(a + 2b \log t), \quad t \geq 1.$$

Compute the mean excess function of X.

Remark: The Log-normal distribution has no closed-form mean excess function. The Benktander type I distribution provides a model having the same type of tail behavior as the Log-normal distribution and a simple mean excess function.

Exercise 3 (6 points) *Expectation for insurance with retention*

Let X be a risk with mean excess loss function e_X.

(a) Show that $\mathbb{E} \max(X - d, 0) = (1 - \mathbb{P}(X > d)) e_X(d)$ for $d > 0$.

(b) Show that $\mathbb{E}(X) = (1 - \mathbb{P}(X > d)) e_X(d) + \mathbb{E}(\min(X, d))$.

Remark. Consider an insured risk X with retention level (Selbstbehalt) d. Then, $Y := \max(X - d, 0)$ is the part that an insurance company has to pay, while $Z := \min(X, d)$ is the part that the insurance holder has to pay.

Hint. You may use without proof the formula $\mathbb{E}Z = \int_0^\infty \bar{F}_Z(t)dt$ valid for any non-negative random variable Z.

Exercise 4 (6 points) *The Poisson point process paradox*

Let $T_0, T_1, T_2, \dots$ be a Poisson point process. Let $s > 0$ and $N(s) = \sum_{i \in \mathbb{N}} 1_{T_i \leq s}$. Compute the distribution function F_{Z_s} of $Z_s := T_{N(s)+1} - T_{N(s)}$ as well as the limit $\lim_{s \to +\infty} F_{Z_s}(t)$ for $t > 0$.