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Summer term 2010
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Risk Theory

Exercise Sheet 8

Due to: 9th June 2010

Exercise 1 (6 points) *Thinning of the binomial distribution*

Let $N, X_1, X_2, \ldots$ be independent such that $N \sim \text{Bin}(n,p)$ and $X_i \sim \text{Bin}(1,q)$ for every $i \in \mathbb{N}$. Show that the total claim size in the collective model $S = X_1 + \ldots + X_N$ satisfies $S \sim \text{Bin}(n, pq)$.

Exercise 2 (6 points) *Total claim size in the collective model with geometric number of claims and exponential claim sizes*

An insurance company uses the collective model for the aggregate claim amount $S = X_1 + \ldots + X_N$ of a portfolio, where $N \sim \text{Geo}(p)$ and $X_i \sim \text{Exp}(\lambda)$ are independent with $p \in (0,1)$ and $\lambda > 0$. Show that

$$\mathbb{P}(S > t) = e^{-\lambda p t}, \quad t \geq 0.$$

Exercise 3 (6 points) *Expectation and variance of the negative binomial distribution*

Let $N \sim \widetilde{\text{NB}}(\gamma, p)$ for some $\gamma > 0$, $p \in (0,1)$. Show by using generating functions that $\mathbb{E}N = \gamma(1-p)/p$ and $\operatorname{Var} N = \gamma(1-p)/p^2$.

Exercise 4 (6 points) *Tails of the log-normal distribution are lighter than power-law tails*

Let X be a random variable with standard log-normal distribution, i.e., $X = e^Y$, where $Y \sim N(0,1)$ is standard normal. Show that for every $\beta > 0$ there exists a $c > 0$ such that $\mathbb{P}[X > t] \leq c t^{-\beta}$ for all $t > 0$.

Exercise 5 (6 points) *Maxima of independent Gaussian random variables*

Let $X_1, X_2, \ldots$ be independent standard normal random variables. Define $M_n = \max\{X_1, \ldots, X_n\}$. Show that for every $t \in \mathbb{R}$,

$$\lim_{n\to\infty} \mathbb{P}\left[M_n \leq \sqrt{2\log n} + \frac{-1/2 \log\log n - \log(2\sqrt{\pi}) + t}{\sqrt{2\log n}}\right] = e^{-e^{-t}}.$$

Hint. Use the formula $\mathbb{P}[X_1 > t] \sim \frac{1}{t}\frac{1}{\sqrt{2\pi}} e^{-t^2/2}$ as $t \to \infty$. You may also use that the relation $\lim_{n\to\infty} x_n = x$ implies that $\lim_{n\to\infty}(1 + \frac{x_n}{n})^n = e^x$.