Supplement for Exercise 3, Sheet 2

It holds:

$$\sum_{j=m}^{k-1} \binom{j-1}{m-1} = \binom{k-1}{m}, \quad m \in \mathbb{N}, \quad k > m.$$

Proof. (i) Right hand side: There are $\binom{k-1}{m}$ possibilities to choose m elements from $\{1, ..., k-1\}$.

(ii) Left hand side: Let $j \in \{m, ..., k-1\}$. There are $\binom{j-1}{m-1}$ possibilities such that the number j is the largest number of the chosen elements from (i).

(iii) Obviously, the events in (ii) are disjoint. Therefore, the number of possibilities can be summed up for $j = m, ..., k-1$: $\sum_{j=m}^{k-1} \binom{j-1}{m-1}$.

$$\overset{(i)-(iii)}{\Longrightarrow} \sum_{j=m}^{k-1} \binom{j-1}{m-1} = \binom{k-1}{m}, \quad m \in \mathbb{N}, \quad k > m.$$

□