

Let $(Y_n)_{n \geq 1}$ iid uniform on $\{1, \dots, 6\}$ (result of n -th throw). Furthermore define recursively

$$\begin{aligned} X'_{n+1} &= X'_n \cup \{Y_{n+1}\} \\ X_0 &= \emptyset \end{aligned}$$

That is, X'_n denotes the set of observed numbers at time n . In particular, we have $X_n = |X'_n|$. For $i_0, \dots, i_{n+1} \in \{0, \dots, 6\}$ with $\mathbb{P}(X_n = i_n, \dots, X_0 = i_0) > 0$ we obtain

$$\begin{aligned} & \mathbb{P}(X_{n+1} = i_{n+1} | X_n = i_n, \dots, X_0 = i_0) \\ &= \frac{P(X_{n+1} = i_{n+1}, X_n = i_n, \dots, X_0 = i_0)}{\mathbb{P}(X_n = i_n, \dots, X_0 = i_0)} \\ &= \sum_{\substack{A \subset \{1, \dots, 6\} \\ |A|=i_n}} \sum_{b \in \{1, \dots, 6\}} \frac{\mathbb{P}(X'_n = A, Y_{n+1} = b, |A \cup \{b\}| = i_{n+1}, X_{n-1} = i_{n-1}, \dots, X_0 = i_0)}{\mathbb{P}(X_n = i_n, \dots, X_0 = i_0)} \\ &= \sum_{\substack{A \subset \{1, \dots, 6\} \\ |A|=i_n}} \sum_{b \in \{1, \dots, 6\}} \frac{\mathbb{P}(Y_{n+1} = b, |A \cup \{b\}| = i_{n+1}) \mathbb{P}(X'_n = A, X_{n-1} = i_{n-1}, \dots, X_0 = i_0)}{\mathbb{P}(X_n = i_n, \dots, X_0 = i_0)} \\ &= \sum_{\substack{A \subset \{1, \dots, 6\} \\ |A|=i_n}} \frac{\left(\mathbf{1}_{i_{n+1}=i_n} \frac{|A|}{6} + \mathbf{1}_{i_{n+1}=i_n+1} \frac{6-|A|}{6} \right) \mathbb{P}(X'_n = A, X_{n-1} = i_{n-1}, \dots, X_0 = i_0)}{\mathbb{P}(X_n = i_n, \dots, X_0 = i_0)} \\ &= \left(\mathbf{1}_{i_{n+1}=i_n} \frac{i_n}{6} + \mathbf{1}_{i_{n+1}=i_n+1} \frac{6-i_n}{6} \right) \end{aligned}$$

Thus $(X_n)_{n \geq 0}$ is a MC with transition matrix $\{p_{ij}\}_{i,j \in \{0, \dots, 6\}} = \left\{ \left(\mathbf{1}_{j=i} \frac{i}{6} + \mathbf{1}_{j=i+1} \frac{6-i}{6} \right) \right\}_{i,j \in \{0, \dots, 6\}}$