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SS 2011
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Markov chains

Problem set 1

Due date: April 19, 2011

Exercise 1

Suppose that weather can be either sunny or rainy. If the weather is sunny on one day, then the next day will be sunny with probability 0.8 and rainy with probability 0.2. If the weather is rainy on one day, then the next day will be rainy with probability 0.5 and sunny with probability 0.5. At day 0 the weather is sunny.

(a) What is the probability that day 3 is a rainy day? (Note: the weather at days 1 and 2 is not specified)

(b) Suppose that day 3 is rainy; what is the probability that day 2 was also rainy?

Exercise 2

Let P and Q be two stochastic $l \times l$-matrices, $l \in \mathbb{N}$.

(a) Show that their product PQ is also a stochastic matrix.

(b) Let $\alpha, \beta \geq 0$ with $\alpha + \beta = 1$. Show that the convex combination $\alpha P + \beta Q$ is also a stochastic matrix

Exercise 3

(a) Let $X_0, X_1, X_2, \ldots$ be independent, identically distributed (iid) random variables with values in $\mathbb{Z}$. Show that $X_0, X_1, X_2, \ldots$ is a Markov chain.

(b) A fair dice is thrown infinitely often. Denote by N_n the number of sixes appearing in the first n casts, $n \in \mathbb{N}$. Put $N_0 = 0$. Show that $N_0, N_1, N_2, \ldots$ is a Markov chain with values in $\mathbb{N}_0$ and compute the transition probabilities.

Exercise 4

Let $X_0, X_1, X_2, \ldots$ and $Y_0, Y_1, Y_2, \ldots$ be two Markov chains with values in $\mathbb{Z}$. Does this imply that $Z_n := X_n + Y_n$ is also a Markov chain?