Junior-Prof. Dr. Z. Kabluchko
Christian Hirsch

SS 2011
April 26, 2011

Markov chains

Problem set 3

Due date: May 3, 2011

Exercise 1

Consider a Markov chain with transition matrix

$$P = \begin{pmatrix} \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} \\ 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} \end{pmatrix}.$$

(a) Identify the communicating classes of this Markov chain.

(b) Which of these classes are closed? Which classes are recurrent?

Please provide an explanation, don't just write the answer.

Exercise 2

A fair die is rolled repeatedly. At time n, denote by X_n the time since the most recent six (also, set $X_0 = 0$). For example, if the rolls are $5, 5, 6, 1, 3, 4, 6, 6, 2, 2, 5, \ldots$, then the sequence $X_1, X_2, \ldots$ is $1, 2, 0, 1, 2, 3, 0, 0, 1, 2, 3, \ldots$.

(a) Write down the transition matrix of this Markov chain.

(b) Show that this chain is irreducible.

(c) Show that any state is recurrent.

Exercise 3

The random walk on the infinite binary tree is the following Markov chain. The state space E consists of all sequences of the form $(\varepsilon_1, \ldots, \varepsilon_n)$, where $\varepsilon_i \in \{L, R\}$ and $n \in \{0, 1, \ldots\}$. The case $n = 0$ corresponds to the empty sequence (the root of the tree), called o, which also belongs to E. The transition probabilities are given by $p_{o,(L)} = p_{o,(R)} = 1/2$ and

$$p_{(\varepsilon_1,\ldots,\varepsilon_n),(\varepsilon_1,\ldots,\varepsilon_n,L)} = p_{(\varepsilon_1,\ldots,\varepsilon_n),(\varepsilon_1,\ldots,\varepsilon_n,R)} = p_{(\varepsilon_1,\ldots,\varepsilon_n),(\varepsilon_1,\ldots,\varepsilon_{n-1})} = 1/3, \qquad n \in \mathbb{N}.$$

All other transition probabilities are 0. Show that this Markov chain is transient.