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SS 2011
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Markov chains

Problem set 5

Due date: May 17, 2011

Exercise 1 [6 points]

Consider the game of tennis when the so-called deuce (Einstand) is reached. If a player wins the next point, he has advantage (Vorteil). On the following point, he either wins the game or the game returns to deuce. Assume that for any point, player A has probability $3/4$ of winning the point and player B has probability $1/4$ of winning the point. This can be considered as a Markov chain with the following 5 states: 1: A has won; 2: B has won; 3: advantage A; 4: deuce; 5: advantage B.

(a) Write down the transition matrix of this Markov chain. Describe the communicating classes. Which of them are closed?

(b) At deuce, find the probability that B will win.

(c) At deuce, find the expected duration of the game.

Exercise 2 [6 points]

A fair die is rolled repeatedly. Let Z_n be the outcome of the n-th roll. Let

$$N = \min\{n \in \mathbb{N} : \exists k < n \text{ such that } Z_k = Z_n\}$$

be the first time at which some outcome is observed twice (not necessarily in two consecutive rolls). Find $\mathbb{E}N$, the expectation of N.

Hint. Consider a Markov chain which counts the number of different outcomes observed so far. If some outcome is observed twice, the Markov chain goes to a state called (say) # which is absorbing.

Exercise 3 [6 points]

A gambler with initial capital $m \in \mathbb{N}$ plays a game repeatedly in which he wins 1 with probability $1/2$ and loses 1 with probability $1/2$. The series of games is over if the gambler has capital 0 (in which case he is ruined) or capital n (in which case he is satisfied and stops to play), where $n \geq m$. Compute the expected duration of the series of games.

Hint. Consider $\ell_m = k_m - k_{m-1}$, where k_m is the expected duration of the series of games given that the initial capital is m.