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Markov chains

Problem set 6

Due date: May 24, 2011

Exercise 1 [6 points]

Consider a Markov chain with transition matrix

$$P = \begin{pmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ 0 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & 0 \end{pmatrix}.$$

(a) Compute all invariant probability measures for this Markov chain.

(b) If the Markov chain starts at state 1, what is the expected time to the first return to state 1?

(c) If the Markov chain starts at state 1, what is the expected number of visits to state 2 before the first return to state 1?

Exercise 2 [6 points]

A drunkard walks on the state space $\{0, 1, \ldots, n\}$, state 0 representing the home and state n representing the bar. If the drunkard is at state i, where $1 \leq i \leq n$, then he goes to state $i+1$ with probability $1/2$, or to state $i-1$ with probability $1/2$. If he is at state 0 (home), he goes to state 1 with probability 1. If the drunkard reaches state n (bar), he stays there. Given that the drunkard starts at state 0 (home), what is the mean hitting time of state n (bar)?

Exercise 3 [6 points]

Consider a Markov chain on the state space $E = \{0, 1, 2, \ldots\}$ with the following transition probabilities. If a Markov chain is in state $i \in E$, then it moves to state $i+1$ with probability $1 - p_i$ or falls down to state 0 with probability p_i. Here, $0 < p_i < 1$ for all $i = 0, 1, \ldots$. Show that this chain is recurrent if and only if $\sum_{i=0}^{\infty} p_i = \infty$.

Exercise 4 [Not compulsory]

A random sequence of convex polygons is generated in the following way. Start with a triangle. In every step, pick two different edges of a current polygon at random, join their midpoints, and pick one of the two resulting smaller polygons at random to be the next in the sequence. Let $X_n + 3$ be the number of edges of the n-th polygon thus constructed ($X_0 = 0$). Find $\mathbb{E}X_n$ and compute all invariant probability distributions of this Markov chain.