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SS 2011
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Markov chains

Problem set 7

Due date: May 31, 2011

Exercise 1 [6 points]

A house contains 5 rooms, A, B, C, D, E. Room C is connected to all other rooms. Also, there is a connection between room A and room B, as well as a connection between room D and room E. There are no further connections. A cat performs a random walk in the house in the following way. It starts in room A. After staying for 1 hour in a room, the cat goes to another room chosen at random from the set of rooms connected to the present location of the cat. All rooms connected to the present location of the cat have equal probabilities to be chosen. For example, if the cat is in B, it goes to A with probability $1/2$ or to C with probability $1/2$.

(a) Compute the unique invariant probability measure of this Markov chain.

(b) Find the expected value of the time of the first return of the cat to A.

(c) Find the expected number of visits to room D before the cat returns to A for the first time.

Exercise 2 [6 points]

(a) It is known that a finite, irreducible Markov chain has exactly 1 invariant probability measure. Construct an example of a finite, not irreducible Markov chain which has more than one invariant probability measure. [Finite means that the state space E is a finite set.]

(b) Construct an example of a Markov chain having no invariant measures (except for the zero measure).

Exercise 3 [6 points]

Consider a Markov chain on the state space $E = \{0, 1, 2, \dots\}$ with transition probabilities $p_{i,i+1} = p$, $p_{i,i-1} = 1 - p$ for $i = 1, 2, \dots$, and $p_{0,1} = p$, $p_{0,0} = 1 - p$. Here, $p \in (0, 1)$.

(a) Show that this chain is recurrent for $p \leq 1/2$ and transient for $p > 1/2$.

(b) Show that this chain is positive recurrent for $p < 1/2$ and null recurrent for $p = 1/2$. In the positive recurrent case, compute the unique invariant probability measure.