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Summer term 2012
20th April 2012

Risk Theory

Exercise Sheet 1

Due to: 27th April 2012

Hint: Exercise sheets should be handed in by a team of exactly two students!

Exercise 1 (6 points)

Let the claim size X be exponentially distributed with parameter $\lambda > 0$, that is the density of X is $f_X(t) = \lambda e^{-\lambda t}$ for $t > 0$.

(a) Compute $\mathbb{P}[X > u]$ for $u > 0$.

(b) Compute $\mathbb{P}[a \leq X \leq b]$ for $0 < a < b < \infty$.

(c) Show that for every $u, v > 0$, $\mathbb{P}[X > u + v | X > u] = \mathbb{P}[X > v]$.

Exercise 2 (6 points)

The claim sizes in the non-life insurance are often modeled by the Pareto or by the Weibull distribution. The density of the Pareto distribution $\text{Par}(\alpha, c)$ with parameters $\alpha, c > 0$ is given by

$$f_{\text{Par}(\alpha,c)}(x) = \frac{\alpha}{c}\left(\frac{c}{x}\right)^{\alpha+1} \mathbb{1}_{(c,\infty)}(x),$$

and the density of the Weibull distribution $\text{W}(r, c)$ with parameters $r, c > 0$ is given by

$$f_{\text{W}(r,c)}(x) = rcx^{r-1}\exp(-cx^r)\mathbb{1}_{[0,\infty)}(x).$$

(a) Compute the expectation and the variance of the $\text{Par}(\alpha, c)$-distribution.

(b) Compute the expectation and the variance of the $\text{W}(r, c)$-distribution.

Hint: For some values of the parameters the results may be infinite.

Exercise 3 (6 points)

Let $X_1, \ldots, X_n$ be independent and identically distributed real-valued random variables with $\mathbb{P}[X_1 > 0] = 1$. Show that

$$\mathbb{E}\left[\frac{X_1}{X_1 + \ldots + X_n}\right] = \frac{1}{n}.$$

Exercise 4 (7 points)

(a) Let $W_1, W_2, \ldots$ be independent random variables having the geometric distribution with parameter $p \in (0, 1]$. Show that the random variable $T_n := W_1 + \ldots + W_n$ satisfies $T_n \sim \text{NB}(n, p)$ and compute the expectation of T_n.

(b) Consider two portfolios. Suppose that the number of claims in the first portfolio is $X_1 \sim \text{NB}(n_1, p)$ and the number of claims in the second portfolio is $X_2 \sim \text{NB}(n_2, p)$.
Assuming that X_1 and X_2 are independent, show that the total number of claims satisfies $X_1 + X_2 \sim \text{NB}(n_1 + n_2, p)$.