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Risk Theory

Exercise Sheet 10

Due to: 6th July 2012

Exercise 1 (6 points)

An insurance police generates during a year N claims with sizes $X_1, X_2, \ldots$. The assumptions of the collective model are satisfied. The distribution of N is given by

n	0	1	2	3
$\mathbb{P}(N=n)$	0.5	0.2	0.2	0.1

The claim sizes X_i have the distribution

x	5	10	20	100
$\mathbb{P}(X=x)$	0.4	0.3	0.2	0.1

(a) Compute the premium for the risk $S = \sum_{k=1}^{N} X_k$ using the variance principle with $\beta = 1$.

(b) Compute the net premium (expected payment of the insurer) under the assumption that the insurer covers only 1 claim in a year and that the insurance holder always reports the largest claim $\max_{k=1,\ldots,N} X_k$ of those which occured. If no claims occured, the insurer pays nothing.

Exercise 2 (3 points)

Let X be a risk having distribution with density $\alpha(1+x)^{-\alpha-1}$, $x > 0$, where $\alpha > 1$ (shifted Pareto distribution). Calculate $\mathbb{E}\max(X - M, 0)$, where $M > 0$.

Exercise 3 (6 points)

Let X be a random variable with finite variance σ^2. Prove that for $x > 0$,

$$\mathbb{P}[X - \mathbb{E}X \geq x] \leq \frac{\sigma^2}{x^2 + \sigma^2} \quad \text{and} \quad \mathbb{P}[|X - \mathbb{E}X| \geq x] \leq \frac{2\sigma^2}{x^2 + \sigma^2}.$$

Under what conditions is the second inequality better than the Chebyshev inequality

$$\mathbb{P}[|X - \mathbb{E}X| \geq x] \leq \frac{\sigma^2}{x^2}?$$

Hint: You can use the Cauchy–Schwarz inequality $(\mathbb{E}(XY))^2 \leq \mathbb{E}(X^2)\mathbb{E}(Y^2)$ without proof. Note that $(x - X) \leq (x - X)1_{X \leq x}$.

Exercise 4 (10 points)

The following table contains the claim sums C_{ik}, the premiums π_i, as well as the Chain-Ladder quotas $\widehat{\gamma}_k$ and the Chain-Ladder factors $\widehat{F}_k$ for the run-off years 0 until 3. It is assumed that all claims are settled in until the end of the third year of occurence.

Occurrence year i	Cumulative claim amounts C_{ik} in run-off year k				premium
	0	1	2	3	π_i
2006(=0)	98	180	240	300	350
2007(=1)	99	204	240		350
2008(=2)	103	576			350
2009(=3)	113				350
Factors $\widehat{F}_k$		3.20	1.25	1.25	
Quotas $\widehat{\gamma}_k$	0.2	0.64	0.8	1.0	

(a) Estimate the future claim sizes of the occurence year 2008 with the Loss Development Method.

(b) Assume that the value $C_{2,1}$ is an outlier which appeared due to some error in the calculations. Which future claim sizes, calculated with the Chain-Ladder Method, are influenced by the outlier $C_{2,1}$?

(c) Estimate the global quota $\widehat{\kappa}$ of the Cape-Cod Method.

(d) Estimate the future claim sizes of the occurence year 2008 with the Cape-Cod Method.