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Risk Theory

Exercise Sheet 11

Due to: 13th July 2012

Please solve 3 problems of your choice. You may also solve the remaining problems, in which case you will receive bonus points for these problems.

Exercise 1 (10 points)

An insurance company wants to estimate the late claims of an insurance portfolio. The estimation has to be based upon the following run-off triangle for claim amounts (Schadenzuwächse) S_{ik}. As always, it is assumed that all claims are completely settled within 4 years.

Occurrence year	Claim amounts S_{ik} in run-off year k				
	k=0	1	2	3	4
2007(=0)	70	30	50	150	100
2008(=1)	60	40	150	50	
2009(=2)	40	60	100		
2010(=3)	80	120			
2011(=4)	100				

(a) Compute the Chain-Ladder factors F_1, F_2, F_3, F_4.

(b) Compute the Chain-Ladder quotas $\gamma_0, \gamma_1, \gamma_2, \gamma_3, \gamma_4$.

(c) Compute the reserve needed to cover the claims which occurred in 2011 (and has not been reported yet).

(d) Estimate the amount that has to be paid in 2013 for claims that date from the occurrence year 2010.

(e) Compute the reserve which is needed to cover the claims which will be reported in 2014.

Exercise 2 (6 points)

For a portfolio of risks the premiums π_i and the current claim sums C_{ik} are known for the years of occurrence 2008 until 2011. Furthermore, there are a-priori estimates α_i for the expected end claim amounts (Endschadenstände) and a-priori estimates γ_k for the run-off pattern for the quotas. It is assumed that all claims are settled until the end of the third year of occurrence.

Occurrence year	Cumulative claim amounts C_{ik} in run-off year k				premium	a priori end claim amounts
	k=0	1	2	3	π_i	α_i
2008(=0)				600	600	800
2009(=1)			400		500	700
2010(=2)		500			400	800
2011(=3)	300				400	900
Quotas γ_k	0.2	0.6	0.8	1.0		

Estimate the reserve needed to cover the claims which will be reported in 2013 using the

(a) Bornhuetter-Ferguson method.

(b) Loss-Development method.

(c) Cape-Cod method.

Exercise 3 (6 points)

Insurance contracts from some portfolio are divided into two classes, A and B. It is known that 70% of all contracts are of type A, the remaining 30% are of type B. Contracts of type A generate in a year a number of claims which is Poisson distributed with parameter 3. Contracts of type B generate in a year a number of claims which is Poisson distributed with parameter 1.

(a) What is the probability that a randomly chosen contract (of unknown type) generates exactly 1 claim during a year?

(b) Some contract generated 1 claim in year 1, 0 claims in year 2, and 2 claims in year 3. What is the probability that this contract is of type A?

(c) Some contract generated 1 claim in year 1, 0 claims in year 2, and 2 claims in year 3. Compute the expected number of claims this contract will generate in year 4.

Exercise 4 (10 points)

An insurance contract generates N claims with claim sizes $X_1, X_2, \ldots$. The assumptions of the collective model are satisfied. The distributions of the random variables N and X_i are given by

k	0	1	2	3
$\mathbb{P}(N = k)$	0.2	0.4	0.3	0.1

t, Euro	10	20	30	40
$\mathbb{P}(X_i = t)$	0.4	0.2	0.2	0.2

(a) Compute the smallest number R (coverage capital) such that the probability that the total claim size $S = X_1 + \ldots + X_N$ is strictly larger than R is smaller than 0.01.

(b) What is the expectation and the variance of the total claim size S?

(c) Suppose that the insurance covers only one claim and the insurance holder reports the largest claim (if there are claims). What is the expected payment of the insurance?

Exercise 5 (8 points)

Consider an insurance contract generating a claim size X which has exponential distribution with parameter $\lambda > 0$. The reinsurer covers the maximum of $X - M$ and 0, if this maximum does not exceed L. Otherwise, the reinsurer covers L (Stop-Loss reinsurance). Let X_S be the insured part and X_R be the reinsured part.

(a) Compute $\mathbb{E}X_R$, $\mathbb{E}X_S$, $\operatorname{Var} X_R$, $\operatorname{Var} X_S$.

(b) Compute the covariance of X_R and X_S.