Junior-Prof. Dr. Z. Kabluchko
Judith Schmidt

Summer term 2012
27th April 2012

Risk Theory

Exercise Sheet 2

Due to: 4th May 2012
Hint: Exercise sheets should be handed in by a team of exactly two students!

Exercise 1 (6 points) *Distribution function of the Erlang distribution*

Let $X \sim \text{Gamma}(k, \lambda)$ be a Gamma–distributed random variable, where $\lambda > 0$ and $k > 0$ is *integer*. Show that the distribution function of X is given by

$$F_X(t) = 1 - \sum_{i=0}^{k-1} \frac{(\lambda t)^i}{i!} e^{-\lambda t}, \quad t \geq 0.$$

Remark. The $\text{Gamma}(k, \lambda)$ distribution with $k \in \mathbb{N}$ is the distribution of the arrival time of the k-th claim in the Poisson point process with intensity λ. It is also called the Erlang distribution.

Exercise 2 (6 points) *The mode of the Gamma distribution*

For a random variable X with a continuous density function f_X, the mode is defined as the value $t_{mode} \in \mathbb{R}$ with the property $f_X(t_{\text{mode}}) = \sup_{t \in \mathbb{R}} f_X(t)$, provided such value exists and is unique. The mode can be interpreted as the "most probable" value of X. Let $X \sim \text{Gamma}(\alpha, \lambda)$, where $\alpha > 1$ and $\lambda > 0$. Compute the mode of X. What happens if $\alpha \in (0, 1]$?

Exercise 3 (6 points) *Convolution property of binomial and Poisson distributions*

(a) Let $X_1 \sim \text{Bin}(n_1, p)$ and $X_2 \sim \text{Bin}(n_2, p)$ be two independent random variables, where $n_1, n_2 \in \mathbb{N}$ and $p \in [0, 1]$. Show that $X_1 + X_2 \sim \text{Bin}(n_1 + n_2, p)$.

(b) Let $Y_1 \sim \text{Poi}(\lambda_1)$ and $Y_2 \sim \text{Poi}(\lambda_2)$ be two independent random variables, where $\lambda_1, \lambda_2 > 0$. Show that $Y_1 + Y_2 \sim \text{Poi}(\lambda_1 + \lambda_2)$.

Exercise 4 (7 points) *Bernoulli claim arrival process*

Let $\xi_1, \xi_2, \ldots$ be independent and identically Bernoulli distributed random variables with parameter $p \in (0, 1]$. Consider the Bernoulli claim arrival process $T_0 = 0$, $T_1 = \min\{k \in \mathbb{N} : \xi_k = 1\}$, $T_2 = \min\{k > T_1 : \xi_k = 1\}$,..., $T_n = \min\{k > T_{n-1} : \xi_k = 1\}$.
Prove that the increments $W_1 := T_1 - T_0, W_2 := T_2 - T_1, \ldots, W_n := T_n - T_{n-1}$ are independent and identically distributed random variables having a geometric distribution with parameter p.