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Risk Theory

Exercise Sheet 3

Due to: 11th May 2012

Exercise 1 (6 points)

Assume that the duration T of a fire is a random variable with distribution $\mathrm{Exp}(\lambda)$, where $\lambda > 0$. The damage $l(t)$ caused by a fire of duration t may be given by $l(t) = a\exp(bt)$ with $a, b > 0$. What is the distribution of the random variable $l(T)$?

Exercise 2 (6 points)

(a) A random variable X has logarithmic distribution with parameter $p \in (0,1)$ (notation: $X \sim \mathrm{Log}(p)$), if

$$\mathbb{P}[X = n] = -\frac{1}{\log(1-p)} \cdot \frac{p^n}{n}, \quad n \in \mathbb{N}.$$

Show that this is indeed a well-defined probability distribution and compute its expectation.

(b) Given a random variable X with values in $\mathbb{N}$, we define $p_n = \mathbb{P}[X = n]$, $n \in \mathbb{N}$. Show that if $X \sim \mathrm{Geo}(p)$ or $X \sim \mathrm{Log}(p)$, then there exist $a, b \in \mathbb{R}$ such that $p_n = (a + \frac{b}{n})p_{n-1}$ for all $n = 2, 3, \ldots$.

Exercise 3 (6 points)

Given a random variable X with values in $\mathbb{N} \cup \{0\}$, we define $p_n = \mathbb{P}[X = n]$, $n \in \mathbb{N} \cup \{0\}$. Prove that if $X \sim \mathrm{Bin}(n,p)$ for some $n \in \mathbb{N}$, $p \in (0,1)$ or $X \sim \widetilde{NB}(c,p)$ for some $c > 0$, $p \in (0,1)$, then there exist $a, b \in \mathbb{R}$ such that $p_n = (a + \frac{b}{n})p_{n-1}$ for all $n \in \mathbb{N}$.

Remark. Recall that we write $X \sim \widetilde{NB}(c,p)$ if $\mathbb{P}[X = n] = \binom{n+c-1}{n} p^c (1-p)^n$ for all $n \in \mathbb{N} \cup \{0\}$.

Exercise 4 (7 points)

Let $T_0, T_1, T_2, \ldots$ be a Poisson point process with intensity λ. For $k, n \in \mathbb{N}$ with $k \leq n$, show that $T_k/T_n \sim \mathrm{Beta}(k, n-k)$, i.e., the density of T_k/T_n is given by

$$f_{T_k/T_n}(t) = \frac{(n-1)!}{(k-1)!(n-k-1)!} t^{k-1}(1-t)^{n-k-1}, \quad t \in [0,1].$$

Hint: Use without proof the following formula: If $Y, Z > 0$ are random variables with joint density $f_{Y,Z}$, then the density of Y/Z is given by $f_{Y/Z}(t) = \int_0^\infty s f_{Y,Z}(st, s)ds$.