# Risk Theory
## Exercise Sheet 5
Due to: 1st June 2012

**Exercise 1** (6 points)

An insurance company uses the collective model for the aggregate claim amount $S = X_1 + ... + X_N$ of a portfolio, where $N \sim \text{Geo}(p)$ and $X_i \sim \text{Exp}(\lambda)$ are independent with $p \in (0, 1)$ and $\lambda > 0$. Show that

$$P(S > t) = \exp(-\lambda p t), \quad t \geq 0.$$

**Exercise 2** (6 points)

Consider two stochastically independent insurance portfolios (collective model is used for both). For the first portfolio, let the number of claims $N \sim \text{Poi}(100)$ and the claim size $X_1 \sim \text{LN}(10, 2)$. For the second portfolio, let the number of claims $L$ be such that $\mathbb{E}L = 1000$, $\text{Var}\, L = 1200$ and the claim size $Y_1 \sim \text{Exp}(1/5000)$.

(a) Calculate the expected value and the variance of the total claim amount of each portfolio.

(b) Calculate the expected value and the variance of the total claim amount after combining both portfolios.

(c) Provide a lower bound for the capital that the insurance company requires to cover the sum of the total claim amounts of both portfolios with a probability of 99% (use Chebyshev's inequality).

**Exercise 3** (6 points)

Let $S = \sum_{i=1}^{N} U_i$ be the total claim amount in the collective model, where $N \sim \text{Poi}(\lambda)$ and $U_i \sim \text{U}(0, 1)$ (uniformly distributed on $[0, 1]$). The premium, denoted by $\Pi$, has to be chosen in such a way that $\Pi = \mathbb{E}(S + R)$, where $R$ is the contribution restitution (Beitragsrückgewähr) defined as follows:

$$R = \begin{cases} \Pi/2, & \text{if no claim has been reported,} \\ (\Pi - U_1)/2, & \text{if a single claim of size } U_1 \text{ has been reported,} \\ 0, & \text{if more than one claim has been reported.} \end{cases}$$

Compute $\Pi$ and give the value of $\Pi$ for $\lambda = 5$.

**Exercise 4** (6 points)

Let $X$ be exponentially distributed with expectation 100. The risk $X$ (exactly one claim of size $X$ occurs per period) is insured with a gross risk premium (Bruttorisikoprämie) of 80. The retention (Selbstbeteiligung) is 50, and the contribution restitution (in case no claim occurs) is 20. Notice that rational behavior of the insurance holders is always assumed.

(a) Starting from what claim size, the insurance holder will report a claim?

(b) Compute the safety loading (i. e. gross risk premium minus net risk premium).

*Definitions.* Contribution restitution is the amount of money an insurance holder obtains when the number of claims he reports in an insurance period is not too large. For example, in Problem 3, the insurance company pays 20 to the insurance holder if no claim is reported.

Retention is the amount of money that an insurance holder has to pay by himself when he reports the claim. For example, in Problem 4, the insurance company pays $\max(X - 50, 0)$ to the insurance holder if a claim is reported.

The net risk premium is the expected value of the payments of the insurance company from a single contract. Note that it is not $\mathbb{E}X$ in Problem 4.