Junior-Prof. Dr. Z. Kabluchko
Judith Schmidt

Summer term 2012
1st June 2012

Risk Theory

Exercise Sheet 6

Due to: 8th June 2012

Exercise 1 (6 points)

An individual claim size has normal distribution with $\mu = 100$ and $\sigma^2 = 9$. The distribution for the number of claims, N, is given in the table below. Determine the probability that aggregate claims exceed 100. Give a reason why modelling losses with the normal distribution (in particular with $\mu = 100$ and $\sigma^2 = 9$) may be reasonable although negative values are possible.

n	0	1	2	3
$P(N = n)$	0.5	0.2	0.2	0.1

You can assume that the values of the normal distribution function are known.

Exercise 2 (6 points)

An insurance company has a portfolio of 1000 independent and identically distributed risks. In each insurance period, a positive claim occurs with probability 0,9 and no claim occurs with probability 0,1 for each risk. If a claim occurs, it is exponentially distributed with expected value 500. The insurance company collects a premium of 70 per period and risk.
Calculate the probability that the total claim amount of the portfolio exceeds the sum of collected premiums using the central limit theorem.

Exercise 3 (6 points)

A risk S in the collective model is modeled by the number N of claims as well as by the claim amounts U_i, $i = 1, 2$, where

k	0	1	2
$\mathbb{P}(N = k)$	0.5	0.3	0.2

u Euro	100	500	1000
$\mathbb{P}(U_i = u)$	0.5	0.3	0.2

The insurance company compensates for the first reported claim in full height, but only one third of the claim size of the second reported claim.

(a) Compute the net premium in the case of the insurance holder reporting every claim.

(b) Compute the net premium in the case where the insurance holder reports the first claim only if its amount is 500 or 1000. If the first claim is not reported, the second claim is fully covered (if it occurs).

(c) What will be the behavior of a rational insurance holder at the end of the insurance period knowing the number of his claims and the claim sizes if it is possible not to report one of the claims. The answer should depend on the number of claims and the claim sizes.

Exercise 4 (6 points)

The distribution of the total claim size in the individual model S may be approximated by the Normal-Power-Approximation (here, $\mu = \mathbb{E}S$, $\sigma^2 = \operatorname{Var} S$, $\gamma = \mathbb{E}(S-\mu)^3/\sigma^3$):

$$\mathbb{P}(S \le t) \approx \Phi\left(\frac{1}{\gamma} \cdot \left(\sqrt{\gamma^2 + 6\gamma \cdot \frac{t-\mu}{\sigma} + 9} - 3\right)\right).$$

(a) Determine the median of the approximation (depending on μ, σ and γ).

(b) Determine the 95%- and the 5%-quantile of the approximation if $\mu = 300$, $\sigma = 7$ and $\gamma = 1$.