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8th June 2012

Risk Theory

Exercise Sheet 7

Due to: 15th June 2012

Exercise 1 (6 points) *Premium calculation using the minimum expected loss principle*

Let X be a random variable and $L(x, P)$ a function of two variables (called the loss function). The premium $\pi(X)$ is defined to be the minimizer of the expected loss $\mathbb{E}L(X, P)$ over all $P \in \mathbb{R}$, that is

$$\pi(X) = \underset{P\in\mathbb{R}}{\operatorname{argmin}}\, \mathbb{E}L(X, P).$$

Compute $\pi(X)$ if a) $L(x, P) = x(x - P)^2$, b) $L(x, P) = (e^{ax} - e^{aP})^2$.

Exercise 2 (10 points) *Properties of the standard deviation principle*

Consider the standard deviation premium calculation principle $\pi(X) = \mathbb{E}X + a\sqrt{\operatorname{Var} X}$, where $a > 0$.

(a) Show that π is homogeneous, that is $\pi(\lambda X) = \lambda\pi(X)$ for every $\lambda > 0$.

(b) Show that π is subadditive, that is $\pi(X + Y) \leq \pi(X) + \pi(Y)$ for all independent random variables X and Y.

(c) Give an example of two independent random variables for which $\pi(X+Y) \neq \pi(X)+\pi(Y)$, thus showing that π is not additive.

(d) For each $a > 0$ construct a random variable X such that $\pi(X) > \max X$.

Exercise 3 (6 points) *First claim arrival in a Beta-mixed Bernoulli process*

Consider the following experiment: first, a random variable $P \sim \operatorname{Beta}(\alpha, \beta)$ is generated. Then, a coin having the probability P of landing heads is tossed infinitely often. Let T_1 be the time at which it lands heads for the first time. Show that

$$\mathbb{P}[T_1 = n] = \frac{B(\alpha + 1, \beta + n - 1)}{B(\alpha, \beta)}, \quad n \in \mathbb{N}.$$

Remark. A random variable X is said to have Beta distribution with parameters $\alpha, \beta > 0$ (notation: $X \sim \operatorname{Beta}(\alpha, \beta)$), if the density of X is given by

$$f_X(t) = \frac{1}{B(\alpha, \beta)} t^{\alpha-1}(1 - t)^{\beta-1} 1_{t\in(0,1)}.$$

Here, $B(\alpha, \beta) = \int_0^1 t^{\alpha-1}(1 - t)^{\beta-1} dt$ is the Beta function.