# Risk Theory
## Exercise Sheet 8
Due to: 22th June 2012

**Exercise 1** (2 points)

Let $X$ be a $G(n, \lambda)$-distributed risk with parameters $n \in \mathbb{N}$ and $\lambda > 0$. Compute the premium for $X$ using the

(a) expected value principle with parameter $\delta > 0$.

(b) standard deviation principle with parameter $\alpha > 0$.

**Exercise 2** (6 points)

Let $U \geq 0$ be a random variable with distribution function $F$ and tail function $\overline{F} = 1 - F$. Let $M > 0$ be a constant and $U_R = \max\{U - M, 0\}$ be the reinsured part of $U$. Show that for $k \in \mathbb{N}$ it holds that

$$E[U_R^k] = \int_M^\infty (x - M)^k \, dF(x).$$

Deduce from this the formula

$$E[U_R] = \int_M^\infty x \, dF(x) - M\overline{F}(M).$$

**Exercise 3** (10 points)

An insurance company wants to estimate the late claims of an insurance portfolio. The estimation has to be based upon the following run-off triangle for claim amounts (Schadenzuwächse) $S_{ik}$.

| Occurrence year | Claim amounts $S_{ik}$ in run-off year $k$ | | | | |
|---|---|---|---|---|---|
| | k=0 | 1 | 2 | 3 | 4 |
| 2002(=0) | 167 | 80 | 76 | 11 | 5 |
| 2003(=1) | 183 | 117 | 18 | 13 | |
| 2004(=2) | 196 | 38 | 25 | | |
| 2005(=3) | 60 | 29 | | | |
| 2006(=4) | 48 | | | | |

(a) Estimate the Chain-Ladder factors $F_1, F_2, F_3, F_4$.

(b) Estimate the Chain-Ladder quotas $\gamma_0, \gamma_1, \gamma_2, \gamma_3, \gamma_4$.

(c) Estimate the reserve needed to cover the claims which occured in 2006 (and has not been reported yet).

(d) Estimate the reserve which is needed to cover the claims which will be reported in 2009.

(e) Estimate the amount that has to be paid in 2008 for claims that date from the occurrence year 2004.

**Exercise 4** (6 points)

A decision maker has utility function $u(x) = \sqrt{x}$, $x \geq 0$. He is given the choice between two random amounts $X$ and $Y$, in exchange for his entire present capital $w > 0$. The probability distributions of $X$ and $Y$ are given by

| $x$ | 400 | 900 |
|---|---|---|
| $\mathbb{P}(X = x)$ | 0.5 | 0.5 |

| $y$ | 100 | 1600 |
|---|---|---|
| $\mathbb{P}(Y = y)$ | 0.6 | 0.4 |

(a) Show that he always prefers $X$ to $Y$.

(b) Determine for which values of $w$ he should decline the offer.

(c) Give an example of a utility function for which he would prefer $Y$ to $X$.