Risk Theory
Exercise Sheet 9
Due to: 29th June 2012

Exercise 1 (6 points)

Consider the stop-loss reinsurance, that is $X = X_S + X_R$, where $X_S = \min\{X, M\}$ is the insured part and $X_R = \max\{X - M, 0\}$ is the reinsured part, for $M > 0$ constant. Let X be exponentially distributed with parameter $\lambda > 0$.

(a) Compute $\mathbb{E}[X_S]$ and $\mathrm{Var}(X_S)$.

(b) Compute $\mathbb{E}[X_R]$ and $\mathrm{Var}(X_R)$.

Exercise 2 (6 points)

Assume there is an algorithm simulating a random variable $U \sim U[0,1]$ with density function $f_U(t) = \mathbb{1}_{[0,1]}(t)$.

(a) Describe an algorithm generating a Pareto-distributed random variable with parameters $\alpha > 0$ and $c > 0$. The density of such random variable is given by $\frac{\alpha}{c}\left(\frac{c}{t}\right)^{\alpha+1}$ for $t \geq c$ and 0 otherwise.

(b) Describe an algorithm generating a Weibull-distributed random variable with parameters $r > 0$ and $c > 0$. The density of such random variable is given by $rct^{r-1}e^{-ct^r}$ for $t \geq 0$ and 0 otherwise.

Exercise 3 (6 points)

Consider the following run-off triangle for cumulative claim amounts (claim sums) C_{ik}:

Occurrence year	Cumulative claim amounts C_{ik} in run-off year k			
	k=0	1	2	3
2003(=0)	10000	25000	45000	65000
2004(=1)	15000	45000	60000	
2005(=2)	20000	55000		
2006(=3)	20000			

Suppose that all claims are settled completely within 4 years. Use the Chain-Ladder Method to

(a) estimate the expected reserve needed to cover claims with the occurence year 2004.

(b) estimate the expected amount to be paid in 2007 and 2008 for claims dating from the occurence year 2005.

Exercise 4 (9 points)

For a portfolio of risks the premiums π_i and the current claim sums C_{ik} are known for the years of occurence 2004 until 2007. Furthermore, there are a-priori estimators α_i for the expected end claim amounts (Endschadenstände) and a-priori estimators γ_k for in the run-off pattern for the quotas. It is assumed that all claims are settled until the end of the third year of occurence.

Occurrence year	Cumulative claim amounts C_{ik} in run-off year k				premium π_i	a priori end claim amount α_i
	k=0	1	2	3		
2004				561	660	600
2005			486		675	650
2006		441			750	700
2007	294				870	750
Quotas γ_k	0.5	0.7	0.9	1.0		

Estimate the reserve for the year for the claims to be paid in 2009 with the

(a) Bornhuetter-Ferguson method.

(b) Loss-Development method.

(c) Cape-Cod method.