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Stochastic networks II

Problem set 10

Due date: July 17, 2012

Exercise 1

Let $X \subset \mathbb{R}^2$ be a homogeneous Poisson process with intensity λ. For $k \geq 1$ denote by $\widetilde{G}(X, k)$ the directed graph with vertex set X and where an edge is drawn from x to y if y is one of the k nearest neighbors of x in X. Furthermore denote by $G(X, k)$ the (undirected) graph where an edge is drawn between x and y if there exists a directed edge from x to y in $\widetilde{G}(X, k)$ or if there exists a directed edge from y to x in $\widetilde{G}(X, k)$. Derive integral expressions for the following characteristics and explicitly compute the occurring integrals for $k = 1$.

(a) the expected number of edges pointing to the origin in the graph $\widetilde{G}(X \cup \{o\}, k)$

(b) the expected degree of o in $G(X \cup \{o\}, k)$

(c) $\mathbb{E}(\nu_1(|G(X, k)| \cap [0, 1]^2))$, where $|G(X, k)| \subset \mathbb{R}^2$ denotes the union of all edges in $G(X, k)$.

Hint. Use the Slivnyak-Mecke formula.

Exercise 2

Let $X \subset \mathbb{R}^2$ be a homogeneous Poisson process with intensity λ. For $A, B \subset \mathbb{R}^2$ write $A + B = \{a + b : a \in A, b \in B\}$. Denote $B_1(o) \subset \mathbb{R}^2$ the unit disk in $\mathbb{R}^2$ and by $G^{(1)}(X)$ the graph with vertex set X and where an edge is drawn from x to y if $X([x, y] \oplus B_1(o)) = 2$. Compute the following characteristics.

(a) the expected degree of o in $G^{(1)}(X \cup \{o\})$

(b) $\mathbb{E}(\nu_1(\left|G^{(1)}(X)\right| \cap [0, 1]^2))$, where $\left|G^{(1)}(X)\right| \subset \mathbb{R}^2$ denotes the union of all edges in $G^{(1)}(X)$.

Hint. Use the Slivnyak-Mecke formula.