Stochastic networks II
Problem set 10
Due date: July 17, 2012

Exercise 1
Let $X \subset \mathbb{R}^2$ be a homogeneous Poisson process with intensity $\lambda$. For $k \geq 1$ denote by $\tilde{G}(X, k)$ the directed graph with vertex set $X$ and where an edge is drawn from $x$ to $y$ if $y$ is one of the $k$ nearest neighbors of $x$ in $X$. Furthermore denote by $G(X, k)$ the (undirected) graph where an edge is drawn between $x$ and $y$ if there exists a directed edge from $x$ to $y$ in $\tilde{G}(X, k)$ or if there exists a directed edge from $y$ to $x$ in $\tilde{G}(X, k)$. Derive integral expressions for the following characteristics and explicitly compute the occurring integrals for $k = 1$.

(a) the expected number of edges pointing to the origin in the graph $\tilde{G}(X \cup \{o\}, k)$

(b) the expected degree of $o$ in $G(X \cup \{o\}, k)$

(c) $E(\nu_1(|G(X, k) \cap [0, 1]^2|))$, where $|G(X, k)| \subset \mathbb{R}^2$ denotes the union of all edges in $G(X, k)$.

Hint. Use the Slivnyak-Mecke formula.

Exercise 2
Let $X \subset \mathbb{R}^2$ be a homogeneous Poisson process with intensity $\lambda$. For $A, B \subset \mathbb{R}^2$ write $A + B = \{a + b : a \in A, b \in B\}$. Denote $B_1(o) \subset \mathbb{R}^2$ the unit disk in $\mathbb{R}^2$ and by $G^{(1)}(X)$ the graph with vertex set $X$ and where an edge is drawn from $x$ to $y$ if $X([x, y] \oplus B_1(o)) = 2$. Compute the following characteristics.

(a) the expected degree of $o$ in $G^{(1)}(X \cup \{o\})$

(b) $E(\nu_1(|G^{(1)}(X) \cap [0, 1]^2|))$, where $|G^{(1)}(X)| \subset \mathbb{R}^2$ denotes the union of all edges in $G^{(1)}(X)$.

Hint. Use the Slivnyak-Mecke formula.