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Stochastic networks II

Problem set 8

Due date: July 3, 2012

Exercise 1

For $a > 0$ write $Q_a = [-a/2, a/2]^2$. Show that there exists $n_0 \geq 1$, $c > 0$ and a function $m : \mathbb{Z}^+ \to \mathbb{Z}^+$ such that for all $n \geq n_0$ we have $m(n) \leq cn^2$ and there exist $m(n)$ balls $B_1, \dots, B_{m(n)}$ of radius $\sqrt{n}$ such that every ball B of diameter at least $n/2$ which intersects Q_n contains at least one of the balls $B_1, B_2, \dots, B_{m(n)}$.

Hint. Consider the set of balls centered at points of $\mathbb{Z}^2 \cap Q_{2n}$.

Exercise 2

Let $X \subset \mathbb{R}^2$ be a homogeneous Poisson point process with intensity 1 and denote by $Vor(X)$ the Voronoi tessellation on X. For $a > 0$ subdivide Q_{5a} into 25 subsquares $K_1, \dots, K_{25}$ of side length a and denote by A_a the event that $X(K_i) \geq 1$ for all $1 \leq i \leq 25$

(a) show that if A_a occurs, then the center of every Voronoi cell of $Vor(X)$ which has non-empty intersection with Q_a is contained in Q_{5a}.

(b) show that $\sum_{n=1}^{\infty} \mathbb{P}(A_n^c) < \infty$ and conclude that with probability 1 every bounded set $B \subset \mathbb{R}^2$ intersects only finitely many cells and edges of $Vor(X)$.

*Hint.*Use Borel-Cantelli for part (b).

Exercise 3

Let $G \subset \mathbb{R}^2$ be a locally finite geometric graph and denote by $E_\cup \subset \mathbb{R}^2$ the union of the edges of G. Provide a rigorous proof for the following implications.

(a) Every connected component of $\mathbb{R}^2 \setminus E_\cup$ forms a cell of G (using the definition given in the lecture notes).

(b) Conversely every cell of G forms a connected component of $\mathbb{R}^2 \setminus E_\cup$.