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SS 2012
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Stochastic networks II

Problem set 9

Due date: July 10, 2012

Exercise 1

Let $N \subset \mathbb{R}^d$ be stationary point process with intensity $\lambda \in (0, \infty)$ and denote by $\mathbb{P}_N$ respectively $\mathbb{P}_N^0$ its distribution respectively Palm distribution. Furthermore write $\mathbb{N}^0 = \{\varphi \in \mathbb{N} : \varphi(\{o\}) > 0\}$.

(a) Show $\mathbb{P}_N(\mathbb{N}^0) = 0$.

(b) Show $\mathbb{P}_N^0(\mathbb{N}^0) = 1$.

Exercise 2

Let $X \subset \mathbb{R}^2$ be a homogeneous Poisson process with intensity 1. Denote by $Del(X)$ respectively $Del(X \cup \{o\})$ the Delaunay triangulation on X respectively $X \cup \{o\}$. The goal of this problem is to prove $\mathbb{E}\deg_{Del(X\cup\{o\})}(o) = 6$, where for a graph $G = (V, E)$ and $v \in V$ we write $\deg_G(v)$ for the degree of the vertex v.

(a) Show that for every $a > 0$ we have

$$\mathbb{E}\deg_{Del(X\cup\{o\})}(o) = a^{-2}\mathbb{E} \sum_{S_n \in X\cap[-a/2,a/2]^2} \deg_{Del(X)}(S_n).$$

(b) Show that for every $n \geq 1$ we have

$$\mathbb{E}\left| \frac{1}{6} \sum_{S_n \in X\cap[-n/2,n/2]^2} \deg_{Del(X)}(S_n) - X([-n/2, n/2]^2) \right| \leq 5\mathbb{E}Y_n,$$

where Y_n denotes the number of cells of $Del(X)$ intersecting $\partial[-n/2, n/2]^2$.

(c) Use the ergodic theorem to conclude $\mathbb{E}\deg_{Del(X\cup\{o\})}(o) = 6$.

Exercise 3

Let $r, \lambda > 0$, let $X \subset \mathbb{R}^2$ be a homogeneous Poisson point process with intensity λ and write $X_0 = X \cup \{o\}$.

(a) Compute the distribution of the degree of the vertex o in the r-graph $G(X_0, r)$.

(b) Compute the expected sum of lengths of all edges incident to o in the r-graph $G(X_0, r)$.

(c) Compute the expected (Euclidean) length of $[0,1]^2 \cap G(X,r)$.