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# Methods of Monte Carlo Simulation II
# Exercise Sheet 1

Deadline: Mai 2, 2014 at 11am before the exercises
Please hand in a printed version of your Matlab code and the output of the programs

**Exercise 1** (3)

Consider two gamblers, $A$ and $B$, playing a game where $A$ wins with probability $q \in (0, 1)$ and $B$ wins with probability $1-q$. Gambler $A$ starts with $s_A$ Euro and $B$ starts with $s_B$ Euro. The gambler who wins gets one Euro from his opponent. They repeat the game infinitely many times. Suppose that negative capital of one of the gamblers is possible.
Let $\{X_n\}_{n\geq 0}$ be the stochastic process where $X_n$ denotes the capital in Euro of gambler $A$ after $n$ games. Show that $\{X_n\}_{n\geq 0}$ is a random walk.

**Exercise 2** (3+3+4)

Consider the case of Exercise 1 with $q = 0.6, s_A = 6, s_B = 4$ and let $\{X_n\}_{n\geq 0}$ be defined as above.

a) Write a Matlab program for simulating the first $N$ values of $\{X_n\}_{n\geq 0}$, i.e. $X_1, \ldots, X_N$. Plot one realization for each $N \in \{10, 100, 10000\}$.

b) Let $\tau = \inf\{n \geq 0 : X_n \in \{0, s_A + s_B\}\}$ be the random number of games after which the game ends if negative capital is not possible. Write a Matlab program for estimating $\mathbb{P}(\tau = 8)$ based on 1000 realizations of $\{X_n\}_{n\geq 0}$.
*Hint: For estimating* $\mathbb{P}(\tau = 8)$ *simulate 1000 realizations* $\{X_n^{(1)}\}_{n\geq 0}, \ldots, \{X_n^{(1000)}\}_{n\geq 0}$ *of* $\{X_n\}_{n\geq 0}$ *and compute the corresponding values of* $\tau$ *denoted by* $\tau^{(1)}, \ldots, \tau^{(1000)}$*. Then, estimate the probability* $\mathbb{P}(\tau = 8)$ *by*

$$\widehat{p_8} = \frac{1}{1000} \sum_{i=1}^{1000} \mathbb{I}\left(\tau^{(i)} = 8\right).$$

c) Calculate $\mathbb{P}(\inf\{n \geq 0 : X_n \geq 8\} \leq 4)$ and write a Matlab programm to estimate $\mathbb{P}(\inf\{n \geq 0 : X_n \geq 8\} \leq 4)$ based on 1000 realizations of $\{X_n\}_{n\geq 0}$. Proceed analogously to part b).