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Summer Term 2014

Methods of Monte Carlo Simulation II
Exercise Sheet 10

Deadline: July 24, 2014 at 1pm before the exercises
Please hand in a printed version of your Matlab code and the output of the programs
The solution for exercises concerning reading material 2 will be presented before the exercise class (July 24 at 12pm)

Exercise 1 (2+3)

Let $\{X_t\}_{t\in\mathbb{R}^d}$, $\{Y_t\}_{t\in\mathbb{R}^d}$ be two independent stationary wide sense isotropic random fields. Define the random field $\{Z_t\}_{t\in\mathbb{R}^d}$ by $Z_t = X_t Y_t$ for each $t \in \mathbb{R}^d$. Then, $\{Z_t\}_{t\in\mathbb{R}^d}$ is stationary.

a) Let $\mathbb{E}X_t = \mathbb{E}Y_t = 0$ for each $t \in \mathbb{R}^d$. Show that $\{Z_t\}_{t\in\mathbb{R}^d}$ is stationary wide sense isotropic.

b) Show that $\{Z_t\}_{t\in\mathbb{R}^d}$ is stationary wide sense isotropic.

Exercise 2 (4)

Let $\{X_t\}_{t\in\mathbb{R}^2}$ be a Gaussian random field with

$$\mathrm{Cov}(X(s), X(t)) = \frac{1}{2}\exp(-\|s-t\|^\nu),$$

for $0 < \nu \leq 2$. Simulate $\{X_t\}_{t\in[0,1/2]^2}$ for each $\nu \in \{1/2, 1, 3/2\}$. Chose a step width of $h = 0.01$. Use the Matlab commands `surf` and `contour` for visualization of each realization.

Exercise 3 (4)

Let A_1, A_2, A_3 be events such that $\mathbb{P}(A_i) > 0$ for each $i \in \{1, 2, 3\}$. Show that A_1 and A_3 are conditionally independent given A_2 if and only if $\mathbb{P}(A_3 \mid A_1 \cap A_2) = \mathbb{P}(A_3 \mid A_2)$.