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Summer Term 2014

Methods of Monte Carlo Simulation II
Exercise Sheet 2

Deadline: Mai 8, 2014 at 1pm before the exercises
Please hand in a printed version of your Matlab code and the output of the programs

Exercise 1 (2)

Let $\{X^{(i)}\}_{i=1}^{N}$ be a sequence of i.i.d. random variables with $\mathbb{E}\left(X^{(1)}\right)^2 < \infty$. Show that

$$\widehat{\sigma^2} = \frac{1}{N-1}\left(\sum_{i=1}^{N}\left(X^{(i)}\right)^2 - \frac{1}{N}\left(\sum_{i=1}^{N} X^{(i)}\right)^2\right)$$

is an unbiased estimator for $\operatorname{Var} X^{(1)}$.

Exercise 2 (2+4)

Consider the random walk $\{X_n\}_{n\geq 0}$ with parameter $q = 0.5$ and $X_0 = 0$.

a) Calculate $\gamma(n) = \operatorname{Var} X_n$ for an arbitrary $n \geq 0$.

b) Write a Matlab program for computing the estimators $\widehat{\gamma(1)}, \ldots, \widehat{\gamma(100)}$ for $\gamma(1), \ldots, \gamma(100)$ based on 1000 realizations of $\{X_n\}_{n\geq 0}$. Compute also the corresponding asymptotic 95% confidence intervals $(a(n), b(n))$ for each $\widehat{\gamma(n)}, n \in \{1, \ldots, 100\}$. Plot $\gamma, \widehat{\gamma}, a$ and b in a single figure in order to visualize your results.
Hint: Plot γ at first and use the command `hold on` *before plotting $\widehat{\gamma}, a$ and b.*

Exercise 3 (2+1+4)

Consider the gamblers A and B from Exercise Sheet 1 with $s_A = s_B = 5$ and $q = 0.5$. If one of the gamblers is ruined, the other one gives him 1 Euro and they continue gambling. Let $\{X_n\}_{n\geq 0}$ be the stochastic process where X_n denotes the capital in Euro of gambler A after n games.

a) Write a Matlab program for simulating the first N values of $\{X_n\}_{n\geq 0}$, i.e. $X_1, \ldots, X_N$. Generate 1000 realizations of $\{X_n\}_{n\geq 0}^{1000}$ and plot a histogram of the realizations of X_{n_0} for each $n_0 \in \{5, 50, 100, 1000\}$. Each possible state of $\{X_n\}_{n\geq 0}$ should be represented by a single bin in the histogram.
Hint: Use the command `hist` .

b) Determine the distribution of X_2.

c) Write a Matlab program for computing the estimators $\widehat{p_i}$ of $\mathbb{P}(X_{1000} = i)$ for $i \in \{0, \dots, 10\}$ such that

$$\mathbb{P}(|\mathbb{P}(X_{1000} = i) - \widehat{p_i}| > 0.01) \leq 0.05.$$