Dr. Tim Brereton
Matthias Neumann

ulm university universität
uulm

Methods of Monte Carlo Simulation II
Exercise Sheet 3

Deadline: Mai 15, 2014 at 1pm before the exercises
Please hand in a printed version of your Matlab code and the output of the programs

Exercise 1   (1+1+2+3)

Let $\{X_n\}_{n\geq 0}$ be the process considered in Exercise 3 on Sheet 2 with $s_A = s_B = 2$ and $q = 0.5$.

a) Show that $\{X_n\}_{n\geq 0}$ is a Markov chain.

b) Is $\{X_n\}_{n\geq 0}$ irreducible? Give a reason for your answer.

c) Compute the transition matrix of $\{X_n\}_{n\geq 0}$.

d) Let now X_0 be random with $\mathbb{P}(X_0 = 0) = \mathbb{P}(X_0 = 1) = 0.5$. Write a Matlab program for estimating the distribution of X_{10} based on 10000 realizations of $\{X_n\}_{n\geq 0}$. Does the estimated distribution look similar to the exact distribution of X_{10}?

Exercise 2   (3+3+2)

Consider a house with four rooms A, B, C and D. There are doors between A and B, B and C, C and D, and between A and D. Assume that there is a cat in room A and a mouse in room C. After one hour both independently change the room, each with probability 0.9. In this case the cat and the mouse move to one of the neighboring rooms, to each with the same probability.

a) How can the process of positions of the cat and the mouse be modeled by a Markov chain? Give the transition matrix of the Markov chain.

b) If the cat and the mouse are in the same room, the cat eats the mouse. Note that the cat does not eat the mouse if they meet each other during moving from one room to another one. Write a Matlab program for estimating the expected life time of the mouse. Simulate the Markov chain 10000 times for the estimation.

c) Write a Matlab program for estimating the probability of the event that the cat eats the mouse in room C based on 10000 realizations of the Markov chain.