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Methods of Monte Carlo Simulation II
Exercise Sheet 6

Deadline: June 12, 2014 at 12pm before the exercises
Please hand in a printed version of your Matlab code and the output of the programs

Exercise 1 (2+2+3+3)

Let $\{N_t\}_{t\geq 0}$ be an inhomogenous Poisson process with intensity function $\lambda(t) = e^{-\theta t}, \theta > 0$.

a) Calculate $\mathbb{P}(N_t - N_{t-1} > 0)$ for arbitrary $t \geq 1$.

b) Use the Borel-Cantelli-Lemma in oder to show that

$$N_\infty = \lim_{t\to\infty} N_t \overset{\text{a.s.}}{<} \infty.$$

c) Let $\{T_n\}_{n=1}^{N_\infty}$ be the unordered set of arrival times of $\{N_t\}_{t\geq 0}$. Show that under the under the condition $N_\infty = k$ the random variables $\{T_n\}_{n=1}^{N_\infty}$ are i.i.d with $T_1 \sim \text{Exp}(\theta)$.

d) Let now $\theta = 1/30$. Write a Matlab program for simulating $\{N_t\}_{t\geq 0}$ on $[0, \infty)$. Plot the realization of the path.

Exercise 2 (2+2)

Suppose that cars arrive at a restaurant in accordance with a Poisson process with rate $\lambda = 10$ per hour. The number of people in each car is independent and takes values 1,2,3,4,5 with respective probabilities 1/3, 1/2, 1/12, 1/24, 1/24. Every thousandth guest of the restaurant receives a free meal.

a) Write a Matlab program for estimating the expected time until the first free meal is given. Use a sample size of 1000.

b) The owner of the restaurant comes H hours after the opening, where $H \sim U(0, 300)$. Write a Matlab program for estimating the expected time between the arrival of the owner and the time that the next free meal is given. Use a sample size of 1000.