Methods of Monte Carlo Simulation II
Exercise Sheet 7

Deadline: June 26, 2014 at 1pm before the exercises
Please hand in a printed version of your Matlab code and the output of the programs

Exercise 1  (2+2+2)

Suppose you have \( n \) dogs. Each dog has its own kennel. If one of the dogs goes into its dog kennel it remains there for \( A_1 \) hours, where \( A_1 \sim \text{Exp}(\lambda) \) with \( \lambda > 0 \). If a dog has left its kennel, it returns to it after \( A_2 \) hours, where \( A_2 \sim \text{Exp}(\mu) \) with \( \mu > 0 \). Let all dogs be in their kennels at time \( t = 0 \). The number of dogs outside their kennels at time \( t \), denoted by \( \{X_t\}_{t \geq 0} \), defines a continuous time Markov chain.

a) Let \( \tau \) be the time when the first dog leaves its kennel. Determine the distribution of \( \tau \).
b) Let \( n = 2 \). Determine the generator \( Q \) of the continuous time Markov chain.
c) Let \( n = 2 \). Determine the stationary distribution of this process.

Exercise 2  (3+2)

Consider a population starting with 5 individuals. New individuals are born with rate \( 1/2 \) and die with rate \( 1/4 \). With rate \( 1/100 \), an epidemic plague leads to the death of many individuals. In this case, each individual dies independently of the others with probability \( 1/2 \). Let \( \{X_t\}_{t \geq 0} \) be the process describing the number of individuals in the population at time \( t \).

a) Write a Matlab program for simulating \( \{X_t\}_{t \geq 0} \) for \( t \in [0, 1000] \). Plot one realization of this process.
b) Write a Matlab program for estimating the expected time until the population consists of more than 50 individuals. Use a sample size of 10000.

Exercise 3  (2)

Let \((X_1, \ldots, X_n)\) be multivariate normal. Show that \(X_1, \ldots, X_n\) are independent if and only if \( \text{Cov}(X_i, X_j) = 0 \) holds for all \( 1 \leq i < j \leq n \).