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Summer Term 2014

Methods of Monte Carlo Simulation II
Exercise Sheet 7

Deadline: June 26, 2014 at 1pm before the exercises
Please hand in a printed version of your Matlab code and the output of the programs

Exercise 1 (2+2+2)

Suppose you have n dogs. Each dog has its own kennel. If one of the dogs goes into its dog kennel it remains there for A_1 hours, where $A_1 \sim \text{Exp}(\lambda)$ with $\lambda > 0$. If a dog has left its kennel, it returns to it after A_2 hours, where $A_2 \sim \text{Exp}(\mu)$ with $\mu > 0$. Let all dogs be in their kennels at time $t = 0$. The number of dogs outside their kennels at time t, denoted by $\{X_t\}_{t \geq 0}$, defines a continuous time Markov chain.

a) Let τ be the time when the first dog leaves its kennel. Determine the distribution of τ.

b) Let $n = 2$. Determine the generator Q of the continuous time Markov chain.

c) Let $n = 2$. Determine the stationary distribution of this process.

Exercise 2 (3+2)

Consider a population starting with 5 individuals. New individuals are born with rate 1/2 and die with rate 1/4. With rate 1/100, an epedemic plague leads to the death of many individuals. In this case, each individual dies independently of the others with probability 1/2. Let $\{X_t\}_{t \geq 0}$ be the process describing the number of individuals in the population at time t.

a) Write a Matlab program for simulating $\{X_t\}_{t \geq 0}$ for $t \in [0, 1000]$. Plot one realization of this process.

b) Write a Matlab program for estimating the expected time until the population consists of more than 50 individuals. Use a sample size of 10000.

Exercise 3 (2)

Let $(X_1, \ldots, X_n)$ be multivariate normal. Show that $X_1, \ldots, X_n$ are independent if and only if $\text{Cov}(X_i, X_j) = 0$ holds for all $1 \leq i < j \leq n$.