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Summer Term 2014

Methods of Monte Carlo Simulation II
Exercise Sheet 8

Deadline: July 3, 2014 at 1pm before the exercises
Please hand in a printed version of your Matlab code and the output of the programs

Exercise 1 (1+2+2+2)

Let $\{Z_n\}_{n=1}^{\infty}$ be a sequence of i.i.d. random variables such that $Z_1 \sim \mathrm{N}(0,1)$. Define the stochastic process $\{X_n\}_{n=1}^{\infty}$ by

$$X_n = \frac{1}{3}\left(Z_n + Z_{n+1} + Z_{n+2}\right), \quad n \geq 2..$$

a) Show that$\{X_n\}_{n=1}^{\infty}$ is a Gaussian process.

b) Let $k, l \geq 1$. Calculate $\mathrm{Cov}\left(X_k, X_l\right)$.

c) Write a Matlab program for simulating $\{X_n\}_{n=1}^{100}$. Plot one realization.

d) Estimate $\mathbb{P}(|X_1 - X_2| < |X_{100} - X_{100000}|)$ by the standard Monte Carlo estimator. Use a sample size of 20000.

Exercise 2 (1+3+2+2)

Let $\{Z_n\}_{n=1}^{\infty}$ be a sequence of i.i.d. random variables such that $Z_1 \sim \mathrm{N}(0,1)$. Define the stochastic process $\{X_n\}_{n=1}^{\infty}$ by $X_1 = Z_1$ and

$$X_n = \frac{1}{2}X_{n-1} + Z_n, \quad n \geq 2.$$

a) Compute $\mathbb{E}X_n$ for arbitrary $n \geq 1$.

b) Let $k, l \geq 1$. Calculate $\mathrm{Cov}\left(X_k, X_l\right)$.

c) Write a Matlab program for simulating $\{X_n\}_{n=1}^{100}$ by simulating a multivariate normal distributed random vector. Plot one realization.

d) Find another way for simulating $\{X_n\}_{n=1}^{100}$. Implement the simulation in Matlab and plot one realization.