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Methods of Monte Carlo Simulation II
Exercise Sheet 9

Deadline: July 18, 2014 at 1pm before the exercises
Please hand in a printed version of your Matlab code and the output of the programs

Exercise 1 (1+1+3)

Let $\{X_t\}_{t\in[0,1]}$ be a fractional Brownian motion with Hurst parameter $H \in (0,1)$.

a) Compute $\mathbb{E}(X_1 \mid X_{\frac{1}{2}} = 1)$.

b) Compute $\text{Var}(X_1 \mid X_{\frac{1}{2}} = 1)$.

c) Let $H = \frac{1}{4}$. Write a Matlab program for simulating $\{X_t\}_{t\in[0,1]}$ with meshsize $h = 1/1000$ under the condition

$$\left(X_{\frac{1}{1000}}, X_{\frac{1}{500}}, X_{\frac{3}{1000}}\right) = \left(\frac{1}{1000}, \frac{1}{500}, \frac{3}{1000}\right).$$

Plot one realization.

Exercise 2 (2+4+3+3+2)

Let $\{W_t\}_{t\in[0,1]}$ be a Brownian motion and let $\sigma > 0$. Define the process $\{X_t\}_{t\in[0,1]}$ by

$$X_t = \exp\left(-\frac{\sigma^2 t}{2} + \sigma W_t\right).$$

a) Compute $\mathbb{E}X_t$ for arbitrary $t \in [0,1]$.

b) Let $n \geq 1$ be an arbitrary integer and let $0 = t_0 \leq t_1 \leq \ldots \leq t_n$ be arbitrary real numbers. Show that $X_{t_1}, \frac{X_{t_2}}{X_{t_1}}, \ldots, \frac{X_{t_n}}{X_{t_{n-1}}}$ are independent Gaussian random variables. Work out the parameters.

c) Let $\sigma = 1$. Write a Matlab program for simulating $\{X_t\}_{t\in[0,1]}$ with meshsize $h = 1/1000$. Plot one realization.
Hint: It is not necessary to compute the covariance matrix.

d) Compute $\text{Cov}(X_t, X_s)$ for arbitrary $t, s \geq 0$.

e) Write a Matlab program for simulating $\{X_t\}_{t\in[0,1]}$ with meshsize $h = 1/1000$ by means of Cholesky-decomposition of the covariance matrix.