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Risk Theory

Exercise Sheet 1

Due to: May 6, 2014

Note: Please submit exercise sheets in couples!

Problem 1 (6 points)

The Pareto distribution as well as the Weibull distribution are important in insurance mathematics. The density of a Pareto ($\mathrm{Par}(\alpha, c)$) distributed random variable with parameters $\alpha, c > 0$ is given by

$$f(x) = \frac{\alpha}{c}\left(\frac{c}{x}\right)^{\alpha+1} \mathbb{1}(x \in (c, \infty)).$$

The density of a Weibull ($\mathrm{W}(r, c)$) distributed random variable with parameters $r, c > 0$ is given by

$$f(x) = rcx^{r-1}e^{-cx^r} \mathbb{1}(x \in [0, \infty)).$$

(a) Compute the expected value and the variance of a Pareto distributed random variable with parameters $\alpha > 2, c > 0$.

(b) Compute the distribution function of a Pareto distributed random variable with parameters $\alpha, c > 0$.

(c) Compute the expected value and the variance of a Weibull distributed random variable with parameters $r, c > 0$.

Problem 2 (4 points)

Let the duration T of a fire be an exponential distributed random variable with parameter $\lambda > 0$. The damage $l(t)$, caused by a fire of length t, is given by $l(t) = ae^{bt}$ with $a, b > 0$. Show that the random variable $l(T)$ is Pareto distributed with parameters $\frac{\lambda}{b}$ and a.

Problem 3 (6 points)

(a) Let X be geometric distributed with parameter $p \in (0, 1)$, that is $\mathbb{P}[X = n] = p(1-p)^n, n \in \mathbb{N}_0$. Show that for all $i, j \in \mathbb{N}_0$

$$\mathbb{P}[X \leq i + j | X \geq j] = \mathbb{P}[X \leq i].$$

(b) Let Y be exponential distributed with parameter $\lambda > 0$. Show that for all $t, s \geq 0$

$$\mathbb{P}[Y > t + s | Y > t] = \mathbb{P}[Y > s].$$

(c) Let $Z_1 \sim \text{Exp}(\beta)$, $Z_2 \sim \text{Exp}(\gamma)$, $\beta, \gamma > 0$ be independent. Show that the minimum of these two random variables is also exponential distributed.

Problem 4 (4 points)

(a) Let X be Poisson distributed with parameter $\lambda > 0$. Calculate the moment-generating function $m_X(t) = \mathbb{E}[e^{tX}], t \in \mathbb{R}$.

(b) Let $X_1, \dots, X_n, n \geq 2$, be i.i.d. random variables with $X_1 \sim \text{Poi}(\lambda), \lambda > 0$. Calculate the moment generating function of the sum $S_n = \sum_{i=1}^{n} X_i$.