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Risk Theory

Exercise Sheet 10

Due to: July 8, 2014

Note: Please submit exercise sheets in couples!

Problem 1 (5 credits)

Let $U_1, \dots, U_n$ be independent and identically distributed random variables with distribution function $F_U(x)$.

(a) Show that for each $m \in \{1, \dots, n\}$ it holds that

$$\mathbb{P}(U_{(m)} \le x) = \sum_{k=m}^{n} \binom{n}{k} F_U^k(x)(1 - F_U(x))^{n-k}.$$

(b) Show that $\mathbb{P}(U_{(1)} \le x) = 1 - (1 - F_U(x))^n$ and $\mathbb{P}(U_{(n)} \le x) = F_U^n(x)$.

Problem 2 (7 credits)

Let $U_1, \dots, U_n$ be independent and identically distributed random variables with distribution function $F_U(x)$. Show that for each $m \in \{1, \dots, n\}$ it holds that

$$\mathbb{P}(U_{(m)} \le x) = \frac{n!}{(n-m)!(m-1)!} \int_0^{F_U(x)} y^{m-1}(1-y)^{n-m} dy.$$

Problem 3 (6 credits)

Consider the collective model of risk theory. Show the following:

(a) For $N \sim \mathrm{Poi}(\lambda)$, $\lambda > 0$ and $U_i \sim \mathrm{Exp}(\delta)$, $\delta > 0$ it holds that

$$\mathbb{E}[U^n_{(N-k+1)}] = \frac{\lambda^k}{\delta^n (k-1)!} \int_0^1 \exp(-\lambda x)(-\log x)^n x^{k-1} dx.$$

(b) For $N \sim \mathrm{Poi}(\lambda)$, $\lambda > 0$ and $U_i \sim \mathrm{Par}(\alpha, 1)$, $\alpha > 0$ it holds that

$$\mathbb{E}[U^n_{(N-k+1)}] = \frac{\lambda^k}{(k-1)!} \int_0^1 \exp(-\lambda x) x^{k-1-n/\alpha} dx, \ n < \alpha k.$$

Problem 4 (6 credits)

Show that the following number R of multiplications is necessary for the computation of $p_k = \mathbb{P}(X^{ind} = k), k = 0, \ldots, m$.

(a) Theorem 4.5.1: $R = \mathcal{O}(b + abk^2)$.

(b) Corollary 4.5.1: $R = \mathcal{O}(b + abk)$.

Problem 5 (6 credits)

The distribution of the total claim size in the individual model S may be assessed by the Normal-Power-Approximation. Here, $\mu = \mathbb{E}[S], \sigma^2 = \mathrm{Var}\,(S), \gamma = \mathbb{E}[S-\mu]^3/\sigma^3$:

$$\mathbb{P}(S \le t) = \Phi\left(\frac{1}{\gamma}\left(\sqrt{\gamma^2 + 6\gamma\frac{t-\mu}{\sigma} + 9} - 3\right)\right).$$

(a) Determine the median of the approximation (depending on μ, σ and γ).

(b) Determine the 95%- and the 5%-quantile of the approximation if $\mu = 300, \sigma = 7$ and $\gamma = 1$.